Exercises from Section 2.5

Exercise 21. The only difficulty here is that the exercise is **too easy**. It is a **specific question** solved by **general principles**.

$$\lim_{x\to 1^+} (2x+4) = 6.$$

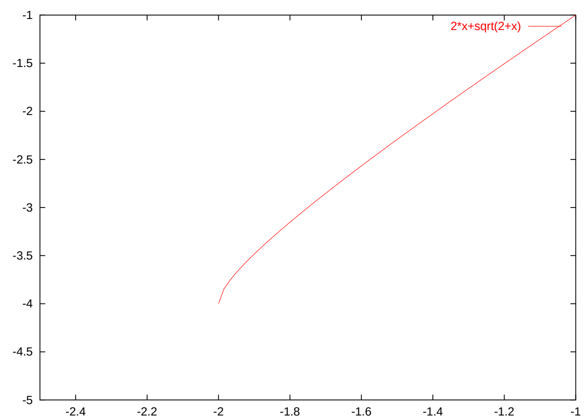
What else could it be? The expression $2x+4$ defines a **continuous function** (defined elsewhere in the section), so the value **at** $x=1$ is the limit as x **approaches** 1 **in any manner**. If the **unqualified** limit exists, both **one-sided** limits also exist and all three limits are equal.

That is (unqualified) limits generalize the values of continuous functions by allowing you to give values at points where a function **just misses** being continuous, and one-sided limits generalize unqualified limits by further restricting the nearby values to be considered. The broader notation is still allowed, and has the expected interpretation, in cases covered by a narrower notation.

Exercise 31.

$$\lim_{x\to -2^+} \left(2x+\sqrt{2+x}\right) = -4.$$

Again, this is just the value of the expression at the value (-2) that x approaches. However, the one-sided limit is essential in this case because the expression $\left(2x+\sqrt{2+x}\right)$ is only defined for $x \geq -2$. The presence of $\sqrt{2+x}$ means that x must be **very close** to -2 for the function to be close to the limit, but we have not provided the tools to elaborate on this distinction. All we can do is to look at a graph and see evidence that it approaches the expected point.



Exercise 33.

$$\lim_{x\to 1^-} \frac{1+x}{1-x} \text{ does not exist,}$$

since the denominator is small, but the numerator is close to 2 if x is close to 1. The indicated division leads to values **of large absolute value**. The one-sided limit allows us to write

$$\lim_{x\to 1^-} \frac{1+x}{1-x} = +\infty$$

to indicate that function values are large and **positive** as x approaches 1 from below. This is reflected in the shape of the graph as it approaches the **vertical asymptote**. (Although the graph is not included here, you are encouraged to obtain it on your calculator.)

NJ Tax Example. Each line of the definition is a linear function, and linear functions are continuous. The one-sided limits of the function are found by evaluating the formula used on that side at the point being approached. We show that the Tax Schedule function shown in lecture is continuous at $x = 20$, but not at $x = 70$.

$$\lim_{x\to 20^-} NJ(x) = (14)(20) = 280$$

$$\lim_{x\to 20^+} NJ(x) = (17.5)(20) - 70 = 350 - 70 = 280$$

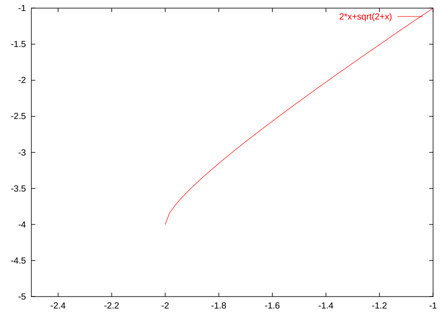
$$\lim_{x\to 70^-} NJ(x) = (24.5)(70) - 420 = 1295$$

$$\lim_{x\to 70^+} NJ(x) = (35)(70) - 1154.5 = 1295.5$$

Exercise 45. Is

$$f(x) = \begin{cases} x+5 & \text{if } x \le 0 \\ -x^2+5 & \text{if } x > 0 \end{cases}$$

contiuous? Again, we examine the graph



It **looks** continuous! To see that it **is**, it suffices to note that each line of the definition is continuous and the one-sided limits

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x+5 = 5$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -x^2+5 = 5$$

are equal.

Exercise 72. We refer to the textbook for the staement of the problem and the graph. The graph shows jump discontinuities at the values $t = 20$, $t = 40$, $t = 60$. Elsewhere the graph appears continuous and decreasing. The continuous behavior corresponds to the **gradual** depletion of inventory through retail sales. The jumps at regular intervals express the given information that 500 reams of paper are **added to** inventory every 20 days.

Exercise 74. The picture here is similar to that in the previous problem, except that the graph is everywhere increasing and the jumps are at two times identified in the statement of the problem as **times of acquisition**. The acquisition of another insitution causes an **abrupt** increase in deposits on the books **immediately** upon acquisition. This is reflected in the graph by having the value at a jump being equal to the limit from the right. (This was also true in the previous problem.)