Exercises from Section 2.6

The method shown in lecture used different notation, than the **four step process** of the textbook. The two approaches share the goal of **finding and simplifying** the difference quotient before allowing the work to be cluttered with the word **limit**. The difference is that the point at which the derivative is to be found was denoted by a in lecture, allowing x to retain its meaning as a **general element of the domain** of the function. In the text, the derivative is being constructed at x, so a new name must be used for the second value entering into the difference quotient. That value is denoted $x + h$, so that the limit that eventually appears has $h \to 0$ instead of $x \to a$. The methods are equivalent, and have different advantages.

As done in lecture, it is clear that a **particular point** is being chosen at which to do the construction. In applications to finding the derivative at a particular point, to construct a **tangent line** (as in exercise 19), there is no x in the formula for the derivative, so it is **immediately** free to be used in the formula for the tangent line. Failure to identify the slope at the given point before attempting to write the equation of the tangent is one of the most annoying mistakes made by students in Calculus courses.

As done in the textbook, the derivative is obtained directly as a function of the same variable used to describe the original function. This emphasizes the derivative as an operation giving new functions for old. When differentiation is done by **formulas** instead of by using limits, there is a benefit to this **economy of notation**. Also, in the **process**, the denominator simplifies to h, and this factor in the numerator will be easily recognized. It greatly simplifies questions that only ask to **find the derivative**; but it requires extra care if the problem continues **and use it to** . . . .

Solutions will be presented here using the notation of the lecture.

Exercise 16. Step through the finding of the slope of the tangent at a general point of the graph of $f(x) = 2x^2 + 5x$.

Solution: We need the slope of the secant line through our **base point** $\big(a,\ f(a)\big)$ and the general point $\big(x,\ f(x)\big)$ on the curve. This is

$$\frac{f(x) - f(a)}{x - a} = \frac{(2x^2 + 5x) - (2a^2 + 5a)}{x - a}$$

The numerator of this expression is

$$\begin{aligned} f(x) - f(a) &= (2x^2 + 5x) - (2a^2 + 5a) \\ &= 2(x^2 - a^2) + 5(x - a) \\ &= 2(x - a)(x + a) + 5(x - a) \\ &= (x - a)(2(x + a) + 5) \end{aligned}$$

Thus,

$$\frac{f(x) - f(a)}{x - a} = 2(x + a) + 5$$

This simplified expression is recognized as continuous, so

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} (2(x + a) + 5) = (2(a + a) + 5) = 4a + 5$$

Exercise 21. To find the tangent to the graph of $f(x) = -1/x$ at $(3, -1/3)$ (which really is of the form $\big(x, f(x)\big)$ with $x = 3$), we begin by finding **and simplifying**

$$f(x) - f(3) = \frac{-1}{x} - \frac{-1}{3} = \frac{-3}{3x} - \frac{-x}{3x} = \frac{x-3}{3x}.$$

Thus,

$$\frac{f(x) - f(3)}{x-3} = \frac{1}{3x},$$

and the limit as $x \to 3$ is $1/9$.

We now have the point $(3, -1/3)$ on the tangent line and a slope $1/9$, so an equation of the line is

$$\frac{y + 1/3}{x - 3} = \frac{1}{9}.$$

Unless a specific form of the equation is required, this is a good place to stop.