Exercises from Section 3.1

These exercises obtain derivatives of functions that can be written as sums of terms that are constants times powers of the independent variable. The rules of this section provide a formal procedure to find derivatives of all such expressions. For each of the exercise a function, usually denoted $f(x)$, is given, and the derivative is to be found. A reason will be given as well as the answer.

Exercise 2. $f(x) = 365$

This is a constant function.

$f'(x) = 0$

Exercise 5. $f(x) = x^{21}$

A power of x.

$f'(x) = 21x^{20}$

Exercise 9. $f(r) = \pi r^2$

π is a constant, so the function is a constant times the square of the variable.

$f'(r) = 2\pi r$

This has an interpretation that will appear in applications. The function $f(r)$ gives the **area** of a circle of radius r. Differentiating gives the perimeter of the circle of radius r. The area between two concentric circles looks like a rectangle of length equal to an average perimeter of the circles and width equal to the difference in radii that has been bent into a circular shape.

Exercise 13. $f(x) = 3\sqrt{x}$

This is a constant multiple of the power function $f(x) = x^{1/2}$. Decreasing the exponent $1/2$ by 1 gives $1/2 - 1 = -1/2$, and the new coefficient is the product of the old coefficient and old exponent.

$f'(x) = 3/2 x^{-1/2}$

Exercise 17. $f(x) = 5x^2 - 3x + 7$

Form a sum of terms, each of which is the given coefficient times the derivative given by the power rule. In practice, these rules are combined into a single process that writes the terms of the answer as the given function is read.

$f'(x) = 10x - 3$

There are two terms in the derivative instead of the three that we started with because one of the terms in the given function was a constant. The derivative of constant is zero, and we usually do not write a term of zero (unless it is the whole answer, as it was in exercise 2).

Exercise 23. $f(x) = \dfrac{x^3 - 4x^2 + 3}{x}$

Begin by dividing **each term** of the numerator by the denominator to get the equivalent expression $f(x) = x^2 - 4x + 3x^{-1}$. Now, one can differentiate term-by-term.

$f'(x) = 2x - 4 - 3x^{-2}$

Note that the only way to differentiate this functions using the tools of this section is by first **preparing the function** to be differentiated.

It is often useful to write answers in a form similar to that used in stating the problem. If the negative exponent were turned back into a denominator of the whole expression, the result would be

$$f'(x) = \frac{2x^3 - 4x^2 - 3}{x^2}.$$

Such a step is usually not required. You should not do it unless it is part of the instructions of a problem or this answer is to be used as the basis of additional work and the simplified form will make that work easier.

Exercise 29. $f(t) = \frac{4}{t^4} - \frac{3}{t^3} + \frac{2}{t}$
Rewrite using negative exponents instead of fractions to get

$$f(t) = 4t^{-4} - 3t^{-3} + 2t^{-1}$$

and differentiate term-by-term.
$f'(t) = -16t^{-5} + 9t^{-4} - 2t^{-2}$
When expressed in the same style as the given expression,

$$f'(t) = -\frac{16}{t^5} + \frac{9}{t^4} - \frac{2}{t^2}.$$

Exercise 33. $f(x) = \frac{2}{x^2} - \frac{3}{x^{1/3}}$
Rewrite as $f(x) = 2x^{-2} - 3x^{-1/3}$ and differentiate term-by-term.
$f'(x) = -4x^{-3} + x^{-4/3}$
The exponent on the last term is $(-1/3) - 1 = (-1/3) - (3/3) = (-1-3)/3$.

Exercises from Section 3.2

The new tool provided by this section if the **product rule**. One exercise was done initially before introducing that rule.

Exercise 5. $f(x) = (3x+1)(x^2-2)$
This can be done using only methods of the previous section by expanding $f(x)$ to $3x^3 + x^2 - 6x - 2$. This shows that the only answer is
$f'(x) = 9x^2 + 2x - 6$
Using the product rule, we have

$$\begin{aligned} f'(x) &= (3x+1)2x + (x^2-2)3 \\ &= 6x^2 + 2x + 3x^2 - 6 \end{aligned}$$

Collecting terms gives the same expression as before.

Exercise 1. $f(x) = (2x)(x^2+1)$
The product rule gives

$$\begin{aligned} f'(x) &= (2x)(2x) + (x^2+1)(2) \\ &= 4x^2 + 2x^2 + 2 \\ &= 6x^2 + 2 \end{aligned}$$

This should be checked by expanding $f(x)$ and differentiating term-by-term.

Exercise 6. $f(x) = (x+1)(2x^2 - 3x + 1)$
Using the product rule, we have

$$\begin{aligned} f'(x) &= (x+1)(4x-3) + (2x^2-3x+1)(1) \\ &= 4x^2 + x - 3 + 2x^2 - 3x + 1 \\ &= 6x^2 - 2x - 2 \end{aligned}$$

This should be checked by expanding $f(x)$ and differentiating term-by-term.

Supplement. The product rule can be extended to products of more than two factors. No formula is need in practice since the following **method** is easy to apply. The result is useful as a guide to understanding the product rule because the emphasizes that each term contains **only one** derivative.

If $f(x) = a(x)g(x)$ with $g(x) = b(x)c(x)$, i.e. $f(x) = a(x)\big(b(x)c(x)\big)$, then

$$\begin{aligned} f'(x) &= a(x)g'(x) + g(x)a'(x) \\ &= a(x)\big(b(x)c'(x) + c(x)b'(x)\big) + b(x)c(x)a'(x) \\ &= a'(x)b(x)c(x) + a(x)b'(x)c(x) + a(x)b(x)c'(x) \end{aligned}$$