## Section 5.1 Exercise 1b
The rule

$$b^x \cdot b^y = b^{x+y}$$

gives

$$3^{-3} \cdot 3^6 = 3^{-3+6} = 3^3 = 27$$

**Exercise 12a** Again, the rule

$$b^x \cdot b^y = b^{x+y}$$

gives

$$y^{-3/2} \cdot y^{5/3} = y^{5/3-3/2} = y^{1/6}$$

**Exercise 18** To solve

$$5^{-x} = 5^3,$$

use the fact that the function $f(t) = 5^t$ is strictly increasing to conclude that equality of the values implies that the exponents are equal. Thus, $-x = 3$, which is easily solved to give $x = -3$.

**Exercise 26** To solve

$$2^{2x} - 4 \cdot 2^x + 4 = 0,$$

introduce $u = 2^x$, which realizes the given equation as

$$u^2 - 4u + 4 = 0.$$

The left side of this equation is $(u-2)^2$, so the only solution is $u = 2$. This says $2^x = 2$, whose only solution is $x = 1$.

## Section 5.4
The general instructions here are to find $f'(x)$ for a given $f(x)$.

**Example from lecture** Starting from

$$e^{2x} = e^{(2x)} = \left(e^x\right)^2,$$

there are two approaches to finding the derivative of $e^{2x}$.

First, as $e^{(2x)}$, it is the exponential function evaluated at $2x$, so the chain rule gives

$$e^{(2x)} \cdot \frac{d}{dx}(2x) = e^{(2x)} \cdot (2) = 2e^{2x}.$$

Second, as $\left(e^x\right)^2$, it is the square of $e^x$, so the general power rule gives

$$2e^x \cdot \frac{d}{dx}\left(e^x\right) = 2e^x \cdot e^x = 2e^{x+x}.$$

Although different paths were taken, the results are the same — as they **must** be, since the definition guarantees that that the derivative is determined by the values of a function.

## Exercise 2

$$f(x) = 3e^x$$
$$f'(x) = 3\frac{d}{dx}\left(e^x\right) = 3e^x$$

## Exercise 6

$$f(x) = 2e^x - x^2$$
$$f'(x) = 2e^x - 2x$$

The derivative of a difference is the difference of the derivatives and the derivative of each term can be written easily.

## Exercise 8

$$f(u) = u^2 \cdot e^{-u}$$
$$f'(u) = u^2(-e^{-u}) + e^{-u}(2u)$$
$$f'(u) = e^{-u}(-u^2 + 2u)$$

The middle line gives the details of the product rule. The last line identifies a common factor of $e^{-u}$ while collecting terms.

## Exercise 10

Given

$$f(x) = \frac{x}{e^x},$$

two methods of calculating the derivative are suggested. First, we can take the expression as it is written and use the quotient rule to obtain

$$f'(x) = \frac{e^x(1) - xe^x}{(e^x)^2}$$

Collecting terms in the numerator gives $e^x(1-x)$, and removing one common factor of $e^x$ leaves

$$f'(x) = \frac{1-x}{e^x}.$$

A second method is to rewrite $f(x) = xe^{-x}$ and use the product rule as in exercise 8. This gives

$$f'(x) = x(-e^{-x}) + e^{-x}(1) = e^{-x}(-x+1).$$

The answers agree.

## Exercise 11

$$f(x) = 3(e^x + e^{-x})$$
$$f'(x) = 3(e^x - e^{-x})$$

There aren't many details to show here. The constant factor of 3 can be extracted while differentiating the other factor. That quantity is a sum that can be differentiated term-by-term. The role of the chain rule in differentiating $e^{-x}$ to get $-e^{-x}$ has been used in previous exercises.

## Exercise 13

$$f(w) = \frac{e^w + 1}{e^w} = 1 + e^{-w}$$

$$f'(x) = -e^{-w} = \frac{1}{e^w}$$

The expression is rewritten in a simpler form before differentiation, and the answer is converted back to a form resembling the original expression for $f(w)$. None of this is necessary. It would not be too difficult to use the quotient rule on the original expression, and the usual instructions in this course include the phrase, "**simplification is not required**". The final form of the answer was chosen only because the given expression was a rational function in $e^w$, and use of the quotient rule would give an answer in that form.

## Exercise 14

Given

$$g(x) = \frac{e^x}{e^x + 1}.$$

This can be written as $g(x) = 1/f(x)$ using the $f$ of Exercise 13, which leads to

$$g'(x) = \frac{-f'(x)}{f(x)^2} = -\left(\frac{e^x}{e^x+1}\right)^2 \left(\frac{-1}{e^x}\right) = \frac{e^x}{(e^x+1)^2}$$

An alternative solution that will be left as an exercise is to use the quotient rule in both exercises 14 and 15 and note the relation between the numerators in the two calculations.

## Exercise 17

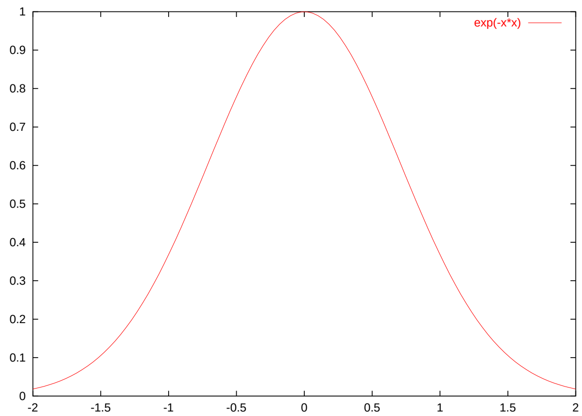
$$f(x) = e^{-x^2}$$

$$f'(x) = e^{-x^2}\frac{d}{dx}\left(-x^2\right) = -2xe^{-x^2}$$

Some of the later exercises ask to find **second derivatives**, but this is a more interesting example than any of those given in that set of exercises, so we find the derivative of our expression for $f'(x)$.

$$f''(x) = -2x(-2x)e^{-x^2} + e^{-x^2}(-2) = (4x^2 - 2)e^{-x^2}$$

This gives information about the infamous **bell curve** that is the graph of (a scaling of) this $f(x)$. The sign of $f'(x)$ is always opposite to that of $x$, so $x = 0$ gives the location of a maximum point, and the second derivative is zero for $x = \pm 1/\sqrt{2} \approx 0.707$. The curve is **concave downward** between the **inflection points** at these values of $x$ and **concave upward** elsewhere. Here is a graph so that you can see these features.



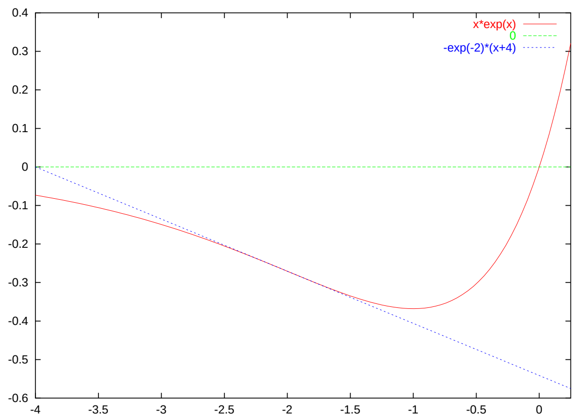
**Exercise 38** The instructions here are to determine the **intervals of concavity** of the graph of $f(x)$, as we did in the epilog to Exercise 17.

$$f(x) = xe^x$$
$$f'(x) = xe^x + e^x(1) = (x+1)e^x$$
$$f''(x) = (x+1)e^x + e^x(1) = (x+2)e^x$$

Since $e^x$ is always positive, the **intervals of concavity** are found by examining the sign of the factor $x+2$. The curve is **concave upward** where this is positive, i.e., for $x > -2$ and **concave downward** where this is negative, i.e., for $x < -2$. A similar analysis of $f'(x)$ shows that there is a minimum at $x = -1$. The function is positive and rapidly increasing for $x > 0$. Also, negative exponential go to zero so rapidly that

$$\lim_{x \to -\infty} xe^x = 0.$$

This gives the **negative** half of the $x$ axis as a horizontal asymptote. Here is a graph of this function with the $x$ axis and the tangent line at the inflection point added to the figure.




You are invited to verify that the tangent line really does meet the $x$ axis exactly at $(-4, 0)$ as shown in the graph.