Section 12.4 In this section, the integral calculus is enlarged to include trigonometric functions.

Exercise 3 Consider

$$\int 3\sin x + 4\cos x\,dx.$$

Linearity shows this to be equal to

$$3\Big(\int \sin x\,dx\Big) + 4\Big(\int \cos x\,dx\Big),$$

where we have isolate integrals that are **known**. Evaluating those integrals gives an answer of

$$-3\cos x + 4\sin x + C.$$

No more need be said since you have a **formula sheet** that identifies the required integrals.

This problem was supplements by finding the **definite integral**

$$\int_0^{\pi/3} 3\sin x + 4\cos x\,dx.$$

Using the **indefinite integral** just obtained, this is

$$-3\cos x + 4\sin x\,\Big|_0^{\pi/3} = \big(-3\cos\frac{\pi}{3} + 4\sin\frac{\pi}{3}\big) - \big(-3\cos 0 + 4\sin 0\big).$$

Now,

$$\cos\frac{\pi}{3} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \quad \cos 0 = 1, \quad \sin 0 = 0,$$

so that the definite integral is

$$\big(-\frac{3}{2} + 2\sqrt{3}\big) - \big(-3 + 0\big) = \frac{3}{2} + 2\sqrt{3}.$$

Exercise 21 Consider

$$\int \sqrt{\cos x}\,\sin x\,dx.$$

This requires the substitution $u = \cos x$, which gives $du = -\sin x\,dx$ (**note the sign**). The integral becomes

$$\int -u^{1/2}\,du = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}\cos^{3/2} x + C$$

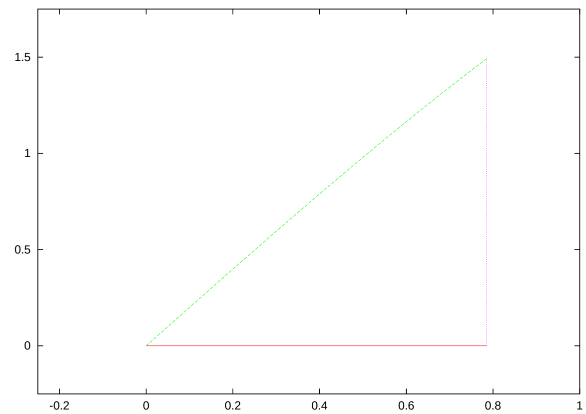
Extra example A simpler example uses the substitution $u = 2x$, which gives $du = 2\,dx$ and $dx = (1/2)\,du$ to evaluate

$$\int \sin 2x\,dx = \big(\sin u\big)\frac{1}{2}\,du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos 2x + C.$$

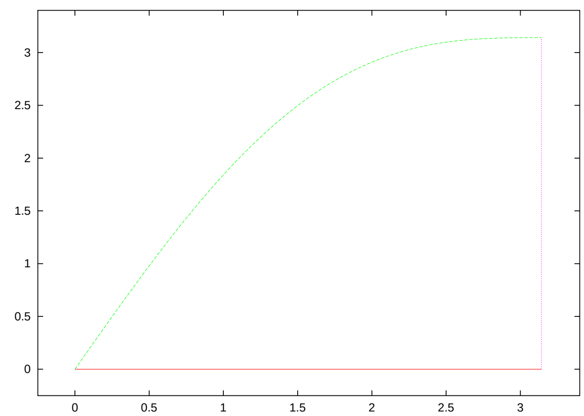
Exercise 34 We are to find the area **under** $y = x + \sin x$ **from** $x = 0$ **to** $\pi/4$. This is just the definite integral

$$\int_0^{\pi/4} x + \sin x \, dx = \frac{x^2}{2} - \cos x \Big|_0^{\pi/4} = \left(\frac{\pi^2}{32} - \frac{\sqrt{2}}{2} \right) - (0 - 1) = \frac{\pi^2}{32} + 1 - \frac{\sqrt{2}}{2} \approx 0.6013 .$$

A graph is



The graph looks very straight over this interval. A more familiar view of the graph uses an interval like $[0, \pi]$ that shows the portion of the graph between two consecutive crossings of $y = x$. Here is that graph



Section 5.3

Exercise 1 This asks for the fate of an investment of $P = 2500$ dollars over a period of $t = 10$ years at a rate of $r = 7\% = 0.07$ per year under **semi-annual compounding**. (The custom is to give interest rates as a percentage to make them seem larger, but our formulas require **numbers**. The conversion uses the **literal meaning** of "per cent", which is **divide by one hundred**.) This gives 20 semi-annual compounding intervals on which the rate is $0.07/2 = 0.035$. When working with compound interest, the **accumulated amount** is found directly. In this example, the amount is

$$A = 2500(1.035)^{20} = 4974.47,$$

which we write to two decimal places because it represents value in dollars.

This is all that was request here, but it is useful to compare this with the result of using the same **nominal rate** with different compounding intervals.

Had the compounding been **annual**, we would have

$$A = 2500(1.07)^{10} = 4917.88,$$

Had the compounding been **continuous**, we would have

$$A = 2500e^{(0.07)(10)} = 5034.34,$$

The difference in these values led to a standard **effective annual yield** that could be used to compare different combinations of nominal rate and compounding interval.