Intervals. The set of numbers **greater than** 1 and **less than or equal to** 5 is written $(1, 5]$ and drawn on a **number line** by putting a round left bracket at 1 and a square right bracket at 5. The **chain of inequalities** expressing the x belongs to this interval is written $1 < x \le 5$. Chaining indicates that **all** inequalities between adjacent quantities are to hold. It is only used when the arrows point in the same direction.

The properties used in working with inequalities are

$$a < b \;\&\; b < c \implies a < c$$
$$a < b \implies a + c < b + c \text{ for all } c$$
$$a < b \;\&\; c > 0 \implies ac < bc$$

Solving inequalities is just like solving equations except that you need to pay attention to the sign of anything that you multiply by.

The special symbol ∞ allows you to write an endpoint that is never reached (so one writes $x < \infty$, never $x \le \infty$).

Absolute values. The **absolute value** of x is defined by

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

For **numbers** this means removing any sign written in front of the number, but you do not know whether a **variable** is positive until you ask, and that is what this definition does.

The absolute value allows you to describe an interval in terms of its **center** and **radius**.

Exponentials. Raising a number to a positive integer power is equivalent to repeated multiplication. This leads to laws of exponents like

$$a^{m+n} = a^m \cdot a^n.$$

Powers of positive numbers are tame and it is **assumed** that arbitrary real powers of a positive base exist and obey the same laws as integer powers. It is possible to **prove** that this is possible, but most people just believe it.

Polynomial equations. The basic algebraic operations of addition, subtraction, multiplication and division are all that you need to solve equations of degree 1.

To solve an equation of degree 2, you can use the quadratic formula (if you remember it). Fortunately, it is also easy to derive the quadratic formula by a process called **completing the square** that may be easier to use that the formula, and has applications to a wide range of problems.

Polynomials of higher degree are not easy to solve unless you recognize a factor.

The Cartesian plane. The projections onto two perpendicular number lines gives a pair of **coordinates** in the plane. Once these number lines, or **axes** are fixed, each point has a unique pair of coordinates and each pair of coordinates determines a unique point. The Pythagorean theorem leads to the formula that the **distance** d between points (x_1, y_1) and (x_2, y_2) satisfies

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2.$$

One important consequence of this distance formula is the equation of a circle.

The equation of a line. The geometry of **similar triangles** is used to show that ratios of **changes** in coordinates are fixed for points on the same line. In particular, the change in y divided by the change in x is called the **slope** of the line. In determining an equation from given geometric information, it often helps to begin by trying to find the slope.

Calculus attempts to extend the methods used to describe straight lines to general curves. The *precalculus* described in this rapid tour of Chapter 1 of the text will return in many forms as we work through Calculus.