Section 2.3 Mathematical models. A **model** is a **function** given by a simple **formula** whose **graph** agrees with something in the **real world** that you want to **study**.

The horizontal axis in the graph is the **independent variable**, which can often be chosen, or controlled in some way. Since our model is a **function**, everything else (the **dependent** variables) will be determined once this independent variable variable is known. In this course, there will usually be only one independent variable and one dependent variable. The dependent variable is usually some result that you want to **predict**, or something else that measures your interest in this process.

The first two examples in the text deal with **time** as the independent variable. The dependent variable is the **cost** of computer security (in example 1) or the **population** enrolled in HMOs (in example 2). In these examples, an attempt is made to find a simple formula that agrees (within experimental error) with the half-dozen or so values that have been reported. The formula can then be used to **explain** the process

measured by the given data or to **predict** the future (barring changes in the process).

Where possible, a **linear** function is used to give a description that is easy to use in calculations. Even if the points don't lie exactly on a line, they may seem to be following one, and there are techniques for selecting a **trend line**.

The next easiest functions to use in computations are **polynomials** and **rational functions** (quotients of polynomials). Pure powers, like $x^{3/4}$ also look simple and are suitable in some cases.

Growth or **decay** models (e.g. population, compound interest or radioactive decay) are based on the exponential function (Chapter 5 o the text — about 2/3 of the way through this course).

Perhaps the most famous mathematical model was proposed by Thomas Malthus (who, like Rutgers University, was born in 1766). His claim was that the growth of products, like manufactured goods or food was linear, but the growth of population was exponential. In the long run, the exponential growth becomes too fast for the linear growth to keep up, and this leads to some sort of disaster. This model is still controversial: to explain continued survival, the models have been refined, and some argue that the refined models refute Malthus' assertions.

Abrupt changes lead to functions defined by cases, like the **piecewise linear** formula used to determine income taxes.

Section 2.4 Limits.

Is

$$\frac{x}{x} = 1$$

?

The answer **should be** yes.

The numerator and the denominator are the same, so **if you can divide** the quotient will be 1.

However, division is not possible if $x = 0$

The answer **is** no.

The domain is an essential part of the definition of a function, even if it is an implicit domain. If two functions have different domains, they must be different.

The concept of **limit** is introduced to allow values to be assigned to missing points in the domain in some cases. The precise definition is somewhat complicated, so our textbook only attempts to state it informally. Essentially, the definition says that

$$\lim_{x\to a} f(x) = L$$

if adding the point (a, L) to the graph of f (after deleting any previous value of $f(a)$) appears to fill in a missing point on the graph of f.

The precise definition allows us to show that, if limits of f and g exist at $x = a$, then limits of algebraic combinations of f and g also exist at $x = a$ and are the same combinations of the limits.

The graphs of polynomials are smooth, so that every point of the graph looks like it belongs. For such functions (called **continuous functions** in Section 2.5, which we will discuss after Section 2.7), limits are found simply by using the original definition of the function.

Sometimes, limits **do not exist**. This happens whenever you can find more than one value that $f(x)$ is

close to in every interval (no matter how small) surrounding $x = a$. An easy example is a function that has one value for $x < a$ and a different value for $x > a$. Since ∞ is not considered to be a number, functions whose graphs have vertical asymptotes at $x = a$ are considered not to have limits at $x = a$. A graph is shown on page 116 with the values of $f(x)$ filling the interval $[-1, 1]$ as x moves through any interval around $x = 0$.

Finding limits. If the graph suggests that a limit exists, how can you find it?

One family of examples is like x/x. If the function is a quotient and the difficulty in finding the limit at $x = a$ is due to a factor of $x - a$ in both numerator and denominator, then that factor of $x - a$ should be removed from both parts, and you should try again to divide.

To deal with a factor of $\sqrt{x} - \sqrt{a}$, you should rationalize the part (numerator or denominator) that contains it. This will lead to factors that are likely to also appear in the other part.