Differentials. It is very tempting to introduce something like dx as a **formal symbol**. For each variable used in a particular discussion, there will be another symbol consisting of the variable's name preceded by a d. If two variables are related, so that one (say y) can be considered as a function of the other (say x), then this function has a derivative denoted by dy/dx. This derivative is an **ordinary** expression, on a par with x and y themselves. When **implicit differentiation** is used, the expression for dy/dx may contain both x and y, but it still reduces to a numerical value when fixed values of the variables are known.

Whatever meaning is given to dx and dy, we expect that

$$dy = \frac{dy}{dx} dx. \qquad (D)$$

The chain rule assures us that

$$dz = \frac{dz}{dy} dy = \frac{dz}{dy}\frac{dy}{dx} dx = \frac{dz}{dx} dx,$$

so these symbols will have a consistent interpretation.

In economic applications, we have seen that derivatives can be used to estimate difference quotients if the **differences** are small. This may be interpreted the symbols like dx to represent the difference between the current value of x and some base value. When this is done for several quantities, (D) gives an **approximate** relation between these differences.

In many cases, this is very useful. However, the approximations given by (D) are not very close, so many of the supposed applications involve exaggerations. Certainly, this should not be seriously used when the function is easily computed. It is never enough to say that one quantity approximates another: some estimate on the size of the difference should also be given to allow you to decide whether the approximation is good enough.

The real value of differential is as formal symbols that organize the information gained from the chain rule. To appreciate that role, it may be useful to consider cases in which they may be given numerical values.