Increasing functions. The word **increasing**, applied to a function f, means that, if $x_0 < x_1$, then $f(x_0) < f(x_1)$.

From this is follows that all **difference quotients** are positive. Since limits of positive quantities cannot be negative, this requires that the derivative be everywhere nonnegative.

A magic result, called the **Mean Value Theorem**, gives an approximate converse to this statement, to the effect that if the derivative is **positive** on an interval, then the function is increasing on that interval.

Decreasing functions. A little reflection shows that $f(x)$ is an decreasing function if and only if $-f(x)$ is an increasing function. This allows statements about increasing functions to be easily transformed into statements about decreasing functions.

Critical Points. If the derivative of a function is not zero at a point, we can always find one nearby point at which the function takes a larger value and another at which the function takes a smaller value.

From this it follows that the largest value or smallest

value of a function can only be taken on at a point where the derivative is zero. (This is a little sloppy. A more precise statement will be given later in this chapter.) Often, these points where the derivative is zero will turn out to give the maximum or minimum value of the function on **some** interval. This property is called a **relative maximum** or **relative minimum**. The points where the derivative is zero are called **critical points** of the function.

This is sort of a **double negative** property: a point that is **not** a critical points, is **not** a relative extremum because the function can be shown to be increasing or decreasing at such a point.