From 135 to 152

Notes on the problem set :

The book used in 151 and 152 is Single Variable Calculus early transcendentals, fourth edition by James Stewart.

Part 1

The problems in part 1 cover limits of the type that will not be repeated in 152. To be able to solve them, you should study sections 2.2, 2.3, 2.5 and 2.6 from Stewart's book.

Compute the following limits or say why they do not exist. Show all work.

$$\lim_{x\to 5^+} \frac{x+2}{x-5} \qquad \lim_{x\to 5^-} \frac{x+2}{x-5} \qquad \lim_{x\to 5^+} \frac{-x+2}{x-5}$$

$$\lim_{x\to 5^-} \frac{-x+2}{x-5} \qquad \lim_{x\to 3} \frac{|x-3|}{x-3} \qquad \lim_{x\to 5^-} \frac{1}{(x-5)^2}$$

$$\lim_{x\to 5^+} \frac{1}{(x-5)^2} \qquad \lim_{x\to 0^+} \ln x$$

$$\lim_{x\to\infty} \frac{\sqrt{x^6+x^4+x^3}}{8x^3+x^2} \qquad \lim_{x\to\infty} \frac{\sqrt{x^5+x^4+x^3}}{2x^3+x^2+x}$$

$$\lim_{x\to\infty} \frac{9x^2+x}{\sqrt[3]{x^6+x^4+x^3}} \qquad \lim_{x\to-\infty} \frac{\sqrt{x^5+x^4+x^3}}{8x^4+x^2}$$

$$\lim_{x\to-\infty} \frac{2x^4-x^3+6}{\sqrt{x^6+x^4+x^3}}$$

$$\lim_{x\to\infty} x-\sqrt{x} \qquad \lim_{x\to\infty} x^3-x^2 \qquad \lim_{x\to\infty} x^2-x^5$$

$$\lim_{x\to-\infty} x^5-x^2 \qquad \lim_{x\to\infty} e^{2x}-e^{x} \qquad \lim_{x\to 0} e^{2x}-e^{x}$$

$$\lim_{x\to\infty} \ln\left(x^2\right)-\ln x \qquad \lim_{x\to\infty} \frac{e^{4x}+2}{5-e^{3x}} \qquad \lim_{x\to-\infty} \frac{e^{4x}+2}{5-e^{3x}}$$

$$\lim_{x\to\infty} \frac{e^{2x}+1}{3+e^{5x}} \qquad \lim_{x\to-\infty} \frac{e^{2x}+1}{3+e^{5x}}$$

$$\lim_{x\to(\pi/2)^+} \tan x \qquad \lim_{x\to(\pi/2)^-} \tan x$$

Part 2

To gain experience with some easy proofs, try to verify the following given facts using **only** the other given facts.

1. Use the quotient rule, the product rule, the fact that $(\sin x)' = \cos x \quad (\cos x)' = -\sin x$ and the trigonometric identidy $\sin^2 x + \cos^2 x = 1$ to verify that :

a. $(\tan x)' = \sec^2 x$ b. $(\cot x)' = -\csc^2 x$
c. $(\sec x)' = \sec x \tan x$ d. $(\csc x)' = -\csc x cotx$

2. Use the law of logarithms, the chain rule, the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and the fact that $(\ln x)' = 1/x$ when $x > 0$, to verify that $(\ln(\tan x))' = (\sin x \cos x)^{-1}$

3. Use the fact that $e^{\ln x} = x$ and implicit differentiation to show that $(\ln x)' = \frac{1}{x}$.
Hint : let $y = \ln x$

4. Here is Theorem 1: $f'(x) = 0$ for all $x \in (a,\ b) \Rightarrow f(x) = C$ for all $x \in (a,\ b)$
Here is Theorem 2 : $f'(x) = g'(x)$ for all $x \in (a,\ b) \Rightarrow f(x) - g(x) = C$ for all $x \in (a,\ b)$
Prove Theorem 2 using Theorem 1.
Hint : Define a new function $h(x) = f(x) - g(x)$

5. Let A and B be **any** positive real numbers such that $A < B$.

Show that $A < \dfrac{A + B}{2} < B$ and $A < \sqrt{AB} < B$.

Part 3

The problems in part 3 use L'Hopital's Rule and applications of $e^{\ln \cdot\cdot}$. Since L'Hopital's Rule requires excellent differentiation skills, make sure that you know how to differentiate **very well**. Then read up on the topic in Stewart's book, and try these problems.
Since in math 152 your instructor will probably spend some time teaching these topics again, if you did **well** on the problems of part 1 and 2, you can feel comfortable in math152 even if you are not able to compute all the limits in this part.

1. Use L'Hopital's Rule to find the following limits. Show all work.

$$\lim_{x\to\infty} \frac{\ln x}{x-1} \qquad \lim_{x\to 1} \frac{\ln x}{x-1} \qquad \lim_{x\to 0^+} x \ln x$$

$$\lim_{x\to\infty} x^3 e^{-x^3} \qquad \lim_{x\to 0} \frac{\sin x - x}{x^3} \qquad \lim_{x\to 0} \frac{1 - e^{-2x}}{\sin x}$$

$$\lim_{x\to\infty} \frac{(\ln x)^3}{x^2} \qquad \lim_{x\to\infty} \frac{\ln \ln x}{x} \qquad \lim_{x\to 0^+} \frac{1 - e^{-2x}}{1 - e^{-5x}}$$

2. Use the fact that if $f(a) > 0$ for some a , then $\lim_{x\to a} f(x) = \lim_{x\to a} e^{\ln f(x)} = e^{\lim_{x\to a} \ln f(x)}$ to compute the following limits:

$$\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x \qquad \lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x \qquad \lim_{x\to 0^+} x^x$$