Answers to Review Problems for Calculus 135 FINAL EXAM

Short answers are listed first. Detailed answers are given following the list of short answers.

Short Answers

1.: $f(x) = e^x - \sin x + 1$

2.: The curve is given by: $y = x^2 - x + 3$

3.:

a. $\frac{1}{3} \ln |x^3 + 3x| + C$

b. $\frac{1}{2}e^{2t} + \frac{1}{3}e^{3t} + C$

c. $\frac{1}{6}$

d. $\frac{1}{6} \ln 3$

e. 11.291528

f. $-5 \cos x + \frac{1}{5} \sin 5x + C$

g. $\frac{1}{9}$

h. $5\frac{1}{2}$

4.: See detailed answers.

5.: a. See detailed answers. b. $\frac{31}{4} = 7.75$ c. $\frac{39}{4} = 9.75$ d. $8\frac{2}{3}$ e. $4\frac{1}{3}$

6.: a. $s(t) = \frac{4}{3}(\sqrt{t})^3$

b. $s(9) = 36$ feet

c. $a(9) = \frac{1}{3}\text{ft/sec}^2$

d. $v(9) = 6\text{ft/sec}$

7.: $P(49) = \$4789\frac{2}{3}$

8.: See detailed answers.

9.: a. $26\frac{2}{3}$

b. $9\frac{1}{6}$

c. see detailed answers for a description of the shaded area.

d. 10:00 A.M.

DETAILED ANSWERS

1. ANSWER: $f(x) = e^x - \sin x + 1$

1. IN DETAIL: We are given $f'(x)$. To find $f(x)$ we need to find the antiderivative of $f'(x)$, that is:

$$f(x) = \int e^x - \cos x dx = e^x - \sin x + C$$

Now, we are given the initial condition that $f(0) = 2$. We use the initial condition: $f(0) = 2$ to solve for C. To do this, we substitute 0 for x in $e^x - \sin x + C$ giving us:

$$e^0 - \sin 0 + C = 2$$

This gives us $1 + C = 2$ which, in turn, gives us $C = 1$. So our function is given by $f(x) = e^x - \sin x + 1$.

2 a. ANSWER: The curve is given by: $y = x^2 - x + 3$

2 a. IN DETAIL: The slope $2x - 1$ is given by the first derivative, so we have $f'(x) = 2x - 1$. The fact that the curve $f(x)$ goes through the point $(1, 3)$ serves as our initial condition, $f(1) = 3$. We proceed as in problem 1 above:

$$f(x) = \int (2x - 1)\, dx = x^2 - x + C$$

Substituting 1 for x and solving for C we get $1 - 1 + C = 3$ which implies that $C = 3$. So our function is given by $f(x) = x^2 - x + 3$.

3 a. ANSWER: $\frac{1}{3} \ln |x^3 + 3x| + C$

3 a. IN DETAIL:

Let $u = x^3 + 3x$. Then $du = (3x^2 + 3)\, dx$ or $\frac{1}{3} du = (x^2 + 1)\, dx$. Now we can rewrite our indefinite integral replacing $x^3 + 3x$ with u and $(x^2 + 1)\, dx$ with $\frac{1}{3}\, du$ to get :

$$\frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln |u| + C$$

Now, this answer is in terms of u so we want to write our answer back in terms of our original variable x, so we get:

$$\frac{1}{3} \ln |x^3 + 3x| + C$$

3 b. ANSWER: $\frac{1}{2} e^{2t} + \frac{1}{3} e^{3t} + C$

3 b. IN DETAIL:

We can write this indefinite integral as a sum of two indefinite integrals to get: $\int e^{2t}\, dt + \int e^{3t}\, dt$. We can take the antiderivative one summand at a time.

Working first with $\int e^{2t}\, dt$ we let $u = 2t$. Then $du = 2\, dt$ or $\frac{1}{2}\, du = dt$. Replacing $2t$ with u and dt with $\frac{1}{2}\, du$ allows us to rewrite $\int e^{2t}\, dt$ as $\frac{1}{2} \int e^u\, du$.

Working next with $\int e^{3t}\,dt$ we let $v = 3t$. Then $dv = 3\,dt$ or $\frac{1}{3}\,dv = dt$. Replacing $3t$ with v and dt with $\frac{1}{3}\,dv$ allows us to rewrite $\int e^{3t}\,dt$ as $\frac{1}{3}\int e^v\,dv$. Now:

$$\frac{1}{2}\int e^u\,du + \frac{1}{3}\int e^v\,dv = \frac{1}{2}e^u + \frac{1}{3}e^v + C$$

Putting our answer in terms of t we get:

$$\frac{1}{2}e^{2t} + \frac{1}{3}e^{3t} + C$$

3 b. ALTERNATE SOLUTION:

The whole integral can be evaluated using the substitution $w = e^t$. Then $dw = e^t\,dt$ and $e^t + e^{2t} = w + w^2$, so the integral is

$$\int w + w^2\,dw = \frac{1}{2}w^2 + \frac{1}{3}w^3 + C$$

Expressing this answer in terms of t gives the same answer as the first solution.

3 c. ANSWER: $\frac{1}{6}$

3 c. IN DETAIL:

Let $u = \ln(x)$. Then $(\ln x)^5 = u^5$, and $du = \frac{1}{x}\,dx$. Since this is a definite integral, and we are using substitution, we now determine the new limits of integration, to reflect the use of $u = \ln(x)$ instead of x. We evaluate u at the original upper limit of integration e to get the new upper limit of integration. The new upper limit of integration is $\ln e = 1$. We evaluate u at the original lower limit of integration 1 to get the new lower limit of integration. The new lower limit of integration is $\ln(1) = 0$. We can now rewrite our definite integral in terms of u, replacing $(\ln x)^5$ with u^5 and $\frac{1}{x}\,dx$ with du giving us :

$$\int_0^1 u^5\,du = \frac{1}{6}u^6\Big|_0^1 = \frac{1}{6} - 0 = \frac{1}{6}$$

If you prefer, you can solve this definite integral by first writing the corresponding indefinite integral:

$$I = \int \frac{(\ln x)^5}{x}\,dx$$

We would use the same u and du as above to get:

$$\int u^5\,du = \frac{1}{6}u^6 + C = \frac{1}{6}\big(\ln|x|\big)^6 + C$$

Now our original problem was a definite integral rather than an indefinite integral so we must evaluate the antiderivative at the limits of integration. The limits of integration that we use, must correspond to our choice of variable used to write our antiderivative. That is, if we use the antiderivative written in terms of the variable x, we must use the limits of integration that correspond to the variable x. If we use the antiderivative written in terms of u, we must use the limits of integration that correspond to the variable u.

So we get:

$$\frac{1}{6}\big(\ln|x|\big)^6\Big|_1^e = \frac{1}{6}\Big(\big(\ln|e|\big)^6 - \big(\ln|1|\big)^6\Big) = \frac{1}{6}(1-0) = \frac{1}{6}$$

3 d. ANSWER: $\frac{1}{6}\ln 3$

3 d. IN DETAIL:

Let $u = 5+2t^3$ and then $du = 6t^2\,dt$ or $\frac{1}{6}\,du = t^2\,dt$. Since this is a definite integral, and we are using substitution, we now determine the new limits of integration. We evaluate u at the original upper limit of integration 2 to get the new upper limit of integration $5+2(2)^3 = 21$. We evaluate u at the original lower limit of integration 1 to get the new lower limit of integration $5+2(1)^3 = 7$. We can now rewrite our definite integral in terms of u replacing $5+2t^3$ with u and t^2dt with $\frac{1}{6}du$ giving us :

$$\frac{1}{6}\int_7^{21}\frac{1}{u}\,du = \frac{1}{6}\ln|u|\Big|_7^{21} = \frac{1}{6}\ln|21| - \frac{1}{6}\ln|7| = \frac{1}{6}\big[\ln(21)-\ln(7)\big] = \frac{1}{6}\ln\frac{21}{7} = \frac{1}{6}\ln 3$$

NOTE: We can drop the absolute value signs since both 21 and 7 are positive.

If you prefer, you can solve this definite integral by first writing the corresponding indefinite integral:

$$I = \int \frac{t^2}{5+2t^3}\,dt$$

We would use the same u and du as above to get:

$$\frac{1}{6}\int\frac{1}{u}du = \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|5+2t^3| + C$$

Now our original problem was a definite integral rather than an indefinite integral so we must evaluate the antiderivative at the limits of integration. The limits of integration that we use, must correspond to our choice of variable used to write our antiderivative. That is, if we use the antiderivative written in terms of the variable t, we must use the limits of integration that correspond to the variable t. If we use the antiderivative written in terms of u, we must use the limits of integration that correspond to the variable u.

So we get:

$$\frac{1}{6}\ln|5+2t^3|\Big|_1^2 = \frac{1}{6}\big[\ln|5+2(2)^3| - \ln|5+2(1)^3|\big] = \frac{1}{6}\big[\ln(21)-\ln(7)\big] = \frac{1}{6}\ln\frac{21}{7} = \frac{1}{6}\ln 3$$

3 e. ANSWER: 11.291528

3 e. IN DETAIL:

Given the two factors in our integrand, there is no suitable choice for u so instead of using substitution, we can multiply the factors in this integrand to give us a sum of terms rather than a product. This gives us: $\int_1^2 2x^{\frac{1}{3}} + x^{\frac{5}{6}} + 2x^2 + x^{\frac{5}{2}}\,dx$. Integrating we get:

$$2\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{11}{6}}}{\frac{11}{6}} + 2\frac{x^3}{3} + \frac{x^{\frac{7}{2}}}{\frac{7}{2}}\Bigg|_1^2 = \frac{3}{2}x^{\frac{4}{3}} + \frac{6}{11}x^{\frac{11}{6}} + 2\frac{x^3}{3} + \frac{2}{7}x^{\frac{7}{2}}\Bigg|_1^2$$

$$= 3\sqrt[3]{2} + \frac{12}{11}\sqrt[6]{2^5} + \frac{16}{3} + \frac{16}{7}\sqrt{2} - \left[\frac{3}{2} + \frac{6}{11} + \frac{2}{3} + \frac{2}{7}\right] = 11.291528$$

3 f. ANSWER: $-5\cos x + \frac{1}{5}\sin 5x + C$

3 f. IN DETAIL:

We can rewrite this indefinite integral as the sum of two indefinite integrals: $5\int \sin x dx + \int \cos 5x\,dx$. Using substitution on the second of these integrals, we let: $u = 5x$ which give us $du = 5\,dx$ or $\frac{1}{5}\,du = dx$. Our indefinite integrals now become:

$$5\int \sin x\,dx + \frac{1}{5}\int \cos u\,du$$

Integrating these indefinite integrals, we get $-5\cos x + \frac{1}{5}\sin u + C$ which, in terms of x, is:

$$-5\cos x + \frac{1}{5}\sin 5x + C$$

3 g. ANSWER: $\frac{1}{9}$

3 g. IN DETAIL:

In this definite integral we can let $u = \sin x$. This gives us $u^8 = \sin^8 x$ and $du = \cos x\,dx$. To rewrite our definite integral in terms of u, we determine limits of integration to reflect our use of $u = \sin x$.

We evaluate $u = \sin x$ at the original upper limit of integration $\frac{\pi}{2}$ to get the new upper limit of integration. The new upper limit of integration is $\sin\frac{\pi}{2} = 1$. We evaluate $u = \sin x$ at the original lower limit of integration 0 to get the new lower limit of integration. The new lower limit of integration is $\sin 0 = 0$. Our definite integral is now written as:

$$\int_0^1 u^8\,du$$

Integrating this we get:

$$\frac{u^9}{9}\Bigg|_0^1 = \frac{1}{9} - 0 = \frac{1}{9}$$

If you prefer, you can solve this definite integral by first writing the corresponding indefinite integral:

$$I = \int \sin^8 x \cos x \, dx$$

We would use the same u and du as above to get:

$$\int u^8 \, du = \frac{1}{9}u^9 + C = \frac{1}{9}\sin^9 x + C$$

Now our original problem was a definite integral rather than an indefinite integral so we must evaluate the antiderivative at the limits of integration. The limits of integration that we use, must correspond to our choice of variable used to write our antiderivative. That is, if we use the antiderivative written in terms of the variable x, we must use the limits of integration that correspond to the variable x. If we use the antiderivative written in terms of u, we must use the limits of integration that correspond to the variable u.

So we get:

$$\frac{1}{9}\sin^9 x \Big|_0^{\frac{\pi}{2}} = \frac{1}{9}\big(\sin^9\left(\frac{\pi}{2}\right) - \sin^9 0\big) = \frac{1}{9}(1-0) = \frac{1}{9}$$

3 h. ANSWER: $5\frac{1}{2}$

3 h. IN DETAIL:

We can use a little algebra to simplify the integrand as follows:

$$\frac{5+2t^3}{t^2} = \frac{5}{t^2} + \frac{2t^3}{t^2} = 5t^{-2} + 2t$$

which gives us:

$$\int_1^2 5t^{-2} + 2t \, dt$$

We can integrate this definite integral term by term to get:

$$-5t^{-1} + t^2\big|_1^2 = \frac{-5}{t} + t^2\Big|_1^2 = \frac{-5}{2} + 4 - \big[-5+1\big] = 5\frac{1}{2}$$

4. ANSWER: We can use a trig identity $\tan x = \frac{\sin x}{\cos x}$ to rewrite our indefinite integral as:

$$\int \frac{\sin x}{\cos x} dx$$

Now we can use substitution as follows:

Let $u = \cos x$. Then $du = -\sin x dx$, or $-du = \sin x dx$. Then our indefinite integral becomes:

$$-\int \frac{1}{u} du$$

Integrating this indefinite integral gives us:

$$-\ln|u| + C$$

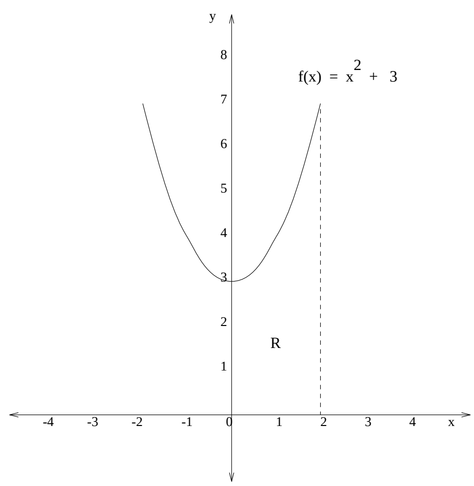
which in terms of x becomes

$$-\ln|\cos x| + C = \ln|\cos x|^{-1} + C = \ln\frac{1}{|\cos x|} + C = \ln|\sec x| + C$$

5. ANSWER a. See below. b. $\frac{31}{4} = 7.75$ c. $\frac{39}{4} = 9.75$ d. $8\frac{2}{3}$ e. $4\frac{1}{3}$

5. IN DETAIL

a. The graph of $f(x) = x^2 + 3$ is the graph of a parabola which is concave up (like a cup) with vertex at the point $(0, 3)$. Your region "R"should be bounded on the bottom by the x axis; it should be bounded on the top by the parabola $y = x^2 + 3$; it should be bounded on the left by the vertical line $x = 0$, i.e. by the y axis, and it should be bounded on the right by vertical line $x = 2$.



b. Our **approximate** area of the region "R" will be given by the sum of the areas of the 4 rectangles whose bases are the 4 subintervals of the interval $[0, 2]$. We begin by determining a partition of the interval $[0, 2]$ into 4 subintervals of equal length. Recall that if we divide an interval $[a, b]$ into "n" subintervals of equal length, each subinterval would have length $\frac{b-a}{n}$. Each of our 4 subintervals, then, will have length $\frac{2-0}{4} = \frac{1}{2}$. The 4 rectangles, then will all have length equal to $\frac{1}{2}$. Our interval $[0, 2]$, then, will be partitioned by the endpoints shown below:

$$0 \underline{\qquad\quad} \tfrac{1}{2} \underline{\qquad\quad} 1 \underline{\qquad\quad} \tfrac{3}{2} \underline{\qquad\quad} 2$$

We need to determine the height of our rectangles in order to compute the area of our rectangles. While our rectangles all have equal length, the heights will vary. The function $f(x) = x^2 + 3$ evaluated at the representative point (left endpoint, right endpoint, or midpoint for example) for a given subinterval gives us the height of the rectangle in that subinterval.

Our representative points are to be the left endpoints ($0, \frac{1}{2}, 1$, and $\frac{3}{2}$) of these subintervals.

This means the height of the rectangle in subinterval $[0, \frac{1}{2}]$ is given by $f\big(0\big)$. The height of the rectangle in subinterval $[\frac{1}{2}, 1]$ is given by $f\big(\frac{1}{2}\big)$. The height of the rectangle in subinterval $[1, \frac{3}{2}]$ is given by $f\big(1\big)$. The height of the rectangle in subinterval $[\frac{3}{2}, 2]$ is given by $f\big(\frac{3}{2}\big)$.

Since the area of a rectangle is length times height, the sum of the areas of these four rectangles becomes:

$$\Big(\frac{1}{2}\Big)f(0) + \Big(\frac{1}{2}\Big)f\big(\frac{1}{2}\big) + \Big(\frac{1}{2}\Big)f(1) + \Big(\frac{1}{2}\Big)f\big(\frac{3}{2}\big)$$

factoring out the $\frac{1}{2}$ we get:

$$\Big(\frac{1}{2}\Big)\Big(f(0) + f\big(\frac{1}{2}\big) + f(1) + f\big(\frac{3}{2}\big)\Big)$$

which gives us:

$$\Big(\frac{1}{2}\Big)\Big(3 + \frac{13}{4} + 4 + \frac{21}{4}\Big) = \frac{31}{4} = 7.75$$

c. To **approximate** the area of region "R" using the right endpoints of the subintervals, we would proceed as in b. above except our representative points would be: $\frac{1}{2}, 1, \frac{3}{2}, 2$.

This means the height of the rectangle in subinterval $[0, \frac{1}{2}]$ is given by $f\big(\frac{1}{2}\big)$. The height of the rectangle in subinterval $[\frac{1}{2}, 1]$ is given by $f\big(1\big)$. The height of the rectangle in subinterval $[1, \frac{3}{2}]$ is given by $f\big(\frac{3}{2}\big)$. The height of the rectangle in subinterval $[\frac{3}{2}, 2]$ is given by $f\big(2\big)$.

The sum of the areas of the four rectangles would become:

$$\Big(\frac{1}{2}\Big)\Big(f\big(\frac{1}{2}\big) + f(1) + f\big(\frac{3}{2}\big) + f(2)\Big)$$

$$= \Big(\frac{1}{2}\Big)\Big(\frac{13}{4} + 4 + \frac{21}{4} + 7\Big) = \frac{39}{4} = 9.75$$

d. To find the **exact** area of the region R, we evaluate the definite integral:

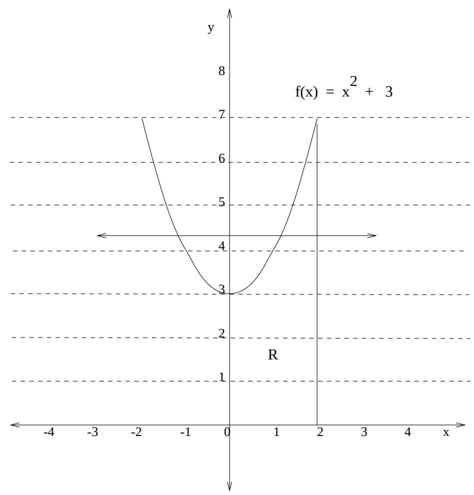
$$\int_0^2 x^2 + 3\,dx = \frac{x^3}{3} + 3x\Big|_0^2 = \big(\frac{8}{3} + 6\big) - (0 + 0) = 8\frac{2}{3}$$

e. The average value of an integrable function $f(x)$ over the interval $[a, b]$ is given by: $\frac{1}{b-a}\int_a^b f(x)\,dx$. The average value of the function $f(x) = x^2 + 3$ over the interval $[0, 2]$ is given by:

$$\frac{1}{2-0}\int_0^2 x^2 + 3\,dt$$

Now, we already computed the definite integral $\int_0^2 x^2 + 3\,dx$ in part d. above, so now we need only multiply the result from d. above by $\frac{1}{2}$. Thus, the average value A of $f(x) = x^2 + 3$ on the interval $[0, 2]$, is $\left(\frac{1}{2}\right)8\frac{2}{3}$ or $4\frac{1}{3}$.

f. Your graph should have a horizontal line which crosses the y axis at $4\frac{1}{3}$. The rectangle bounded below by the x axis, and bounded above by the horizontal line $y = 4\frac{1}{3}$ and bounded on the left by the vertical line $x = 0$ (i.e. the y axis), and bounded on the right by the vertical line $x = 2$ has an area equal to the the area of the region R. To see this, the height of this rectangle is equal to $A = 4\frac{1}{3}$ and the length is equal to 2 (since the base of the rectangle is the interval $[0, 2]$). Now, the area of this rectangle with height $A = 4\frac{1}{3}$ and length 2 is $(2)4\frac{1}{3}$ which equals $8\frac{2}{3}$. This is equal to the exact area of the region R which we computed in d. above.



6 a. ANSWER: $s(t) = \frac{4}{3}\left(\sqrt{t}\right)^3$

6 a. IN DETAIL:

Recall, that the velocity function is the first derivative of the position function. Now, we are given the velocity function and are asked to find the position function. To do this, we must take the antiderivative of the velocity function. That is, the position function $s(t) = \int v(t)dt$.

So we integrate the indefinite integral:

$$s(t) = \int 2\sqrt{t}dt = \int 2t^{\frac{1}{2}}\,dt = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{4}{3}t^{\frac{3}{2}} + C$$

Now we need to find the value of C, so we use the initial condition given in the problem. We are told the initial position of the car is 0 feet. This means when $t = 0$, $s(t) = 0$, or simply $s(0) = 0$. Now $s(0) = \frac{4}{3}0^{\frac{3}{2}} + C = 0$ which implies that $C = 0$ so our position function is given by

$$s(t) = \frac{4}{3}t^{\frac{3}{2}} = \frac{4}{3}\left(\sqrt{t}\right)^3$$

6 b. ANSWER: $s(9) = 36$ feet

6 b. IN DETAIL:

The position of the car at $t = 9$ seconds is $s(9) = \frac{4}{3}(9^{\frac{3}{2}}) = \frac{4}{3}\left(\sqrt{9}\right)^3 = \frac{4}{3}(27) = 36$ feet.

6 c. ANSWER: $a(9) = \frac{1}{3}\text{ft/sec}^2$

6 c. IN DETAIL:

To find the acceleration, we find the second derivative of the position function, or, since we already have the velocity function, we can find the first derivative of the velocity function. The acceleration function is given by :

$$a(t) = s''(t) = v'(t) = t^{-\frac{1}{2}} = \frac{1}{\sqrt{t}}$$

At $t = 9$ acceleration is: $a(9) = \frac{1}{\sqrt{9}} = \frac{1}{3}\text{ft/sec}^2$

6 d. ANSWER: $v(9) = 6\text{ft/sec}$

6 d. IN DETAIL:

The velocity at time $t = 9$ is given by $v(9) = 2\sqrt{9} = 6\text{ft/sec}$.

7. ANSWER: $\$4789\frac{2}{3}$

7. IN DETAIL:

The marginal profit function is the first derivative of the profit function. To find the profit function, we must find the antiderivative of the marginal profit function. To do this we integrate the indefinite integral:

$$p(x) = \int 100 - 0.5\sqrt{x}\,dx = 100x - 0.5\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = 100x - \frac{1}{3}(\sqrt{x})^3 + C$$

Now, we need to solve for the value of "C". To do this, we use the conditions given in the problem, namely that the profit from selling 9 items is 895 dollars. So we know that $p(9) = 100(9) - \frac{1}{3}(\sqrt{9})^3 + C = 895$. From this we get: $891 + C = 895$ which implies that $C = 4$ so our profit function is given by:

$$p(x) = 100x - \frac{1}{3}(\sqrt{x})^3 + 4$$

The profit when $x = 49$ is given by:

$$p(49) = 100(49) - \frac{1}{3}(\sqrt{49})^3 + 4 = 4900 - 114\frac{1}{3} + 4 = 4789\frac{2}{3}$$

8. ANSWER IN DETAIL:

Function	Antiderivative
xe^{2x}	$\left(\frac{x}{2} - \frac{1}{4}\right)e^{2x}$
$\sin^6 x \cos x$	$\frac{\sin^7 x}{7}$
$\frac{\ln x}{x^2}$	$-\left(\frac{\ln x}{x} + \frac{1}{x}\right)$
$-x \sin x + \cos x$	$x \cos x$

Since we are pairing functions with their antiderivatives, it is clear that the function containing terms with $\ln x$ and $\frac{1}{x}$ should be paired with the function containing $\ln x$ and $\frac{1}{x^2}$. Similarly it is clear that the two functions containing terms with e^{2x} should be paired together.

That leaves us with four functions containing terms with $\sin x$ and $\cos x$. To pair them up correctly, we can readily see that the function containing $\sin x$ to the 7^{th} power should be paired with the function containing $\sin x$ to the 6^{th} power. That leaves two functions which will be paired together. Given a pair of functions, the one which is the derivative of the other is in the "Function" column in the table above. The other function in the given pair is the "Antiderivative". To see this, we can take the derivative of the functions in the column marked "Antiderivative". The derivatives of the functions in the column titled "Antiderivative' are in the column titled "Function"as shown below:

$$1. \quad \frac{d}{dx}\left(\left(\frac{x}{2}-\frac{1}{4}\right)e^{2x}\right)=\left(\frac{x}{2}-\frac{1}{4}\right)2e^{2x}+\frac{1}{2}e^{2x}$$

$$=\left(\frac{1}{2}\right)(2)xe^{2x}-\left(\frac{1}{4}\right)(2)e^{2x}+\left(\frac{1}{2}\right)e^{2x}=xe^{2x}-\left(\frac{1}{2}\right)e^{2x}+\left(\frac{1}{2}\right)e^{2x}=xe^{2x}$$

$$2. \quad \frac{d}{dx}\left(\frac{\sin^7 x}{7}\right)=\sin^6 x\frac{d}{dx}(\sin x)=\sin^6 x\cos x$$

$$3. \quad \frac{d}{dx}\left(-\left(\frac{\ln x}{x}+\frac{1}{x}\right)\right)=-\left(\left(\frac{x\left(\frac{1}{x}\right)-\ln x}{x^2}\right)-\frac{1}{x^2}\right)=-\left(\frac{1-\ln x-1}{x^2}\right)=\frac{\ln x}{x^2}$$

$$4. \quad \frac{d}{dx}(x\cos x)=x\frac{d}{dx}(\cos x)+\cos x\frac{d}{dx}(x)=-x\sin x+\cos x$$

9. ANSWER IN DETAIL: Note: The problem reads: The number of products assembled by the average worker **per hour** t hours after starting working at 8 A.M. ... etc." The "per hour" is necessary because this function gives the **rate** at which the average worker assembles the products.

This wording makes the function $f(t)=10t-\frac{5t^2}{2}$ a rate.

9 a. ANSWER: $26\frac{2}{3}$

9 a. IN DETAIL: To find the total number of units the average worker can be expected to assemble in the 4 hours morning shift can be determined by evaluating the definite integral:

$$\int_0^4 10t-\frac{5t^2}{2}\,dt=5t^2-\frac{5t^3}{6}\Bigg|_0^4=80-\frac{320}{6}-(0-0)=26\frac{2}{3}$$

Note: The morning shift extends from the start time 8 A.M. to 12:00 noon. As such, we used 0 as the lower limit of integration because at 8 A.M. $t=0$. We used 4 as the upper limit of integration because the range of t is from 0 to 4 and t = 4 corresponds to 12:00 noon, at which point morning ends.

9 b. ANSWER: $9\frac{1}{6}$

9 b. IN DETAIL:

b. Since 8 A.M. corresponds to $t = 0$, 10:00 A.M. corresponds to $t = 2$ and 11:00 A.M. corresponds to $t = 3$. To find the total number of units the average worker can be expected to assemble between 10:00 and 11:00 A.M. we evaluate the definite integral:

$$\int_2^3 10t - \frac{5t^2}{2}\,dt = 5t^2 - \frac{5t^3}{6}\Big|_2^3 = 45 - \frac{45}{2} - \big(20 - \frac{40}{6}\big) = 9\frac{1}{6}$$

9 c. ANSWER IN DETAIL:

c. The shaded portion of the graph should be bounded on top by the curve of the function $f(t) = 10t - \frac{5t^2}{2}$. It should be bounded on the bottom by the t axis. (We say the t axis instead of the x axis, because f is a function of t rather than a function of x). Now 9:30 A.M. corresponds to $t = 1.5$ and 11:00 A.M. corresponds to $t = 3$. Thus, the graph should be bounded on the left by the vertical line $t = 1.5$ and on the right by the vertical line $t = 3$.

9 d. ANSWER: 10:00 A.M.

9 d. IN DETAIL:

d. The time at which the average worker is expected to be the most efficient, is the time at which his or her rate of assembling units is the greatest. This will correspond to the highest point on the graph of $f(t)$ on the interval $[0, 4]$ which occurs at $t = 2$ or 10:00 A.M. This high point can be recognized in a variety of ways:

1. You can graph the function $f(t)$ and determine from the graph, the value of t at which the graph attains a high point in $[0, 4]$.

2. If you remember your precalculus, you would recognize that the graph of $f(t)$ is a parabola which is concave down so the high point occurs at the vertex. We know that the vertex is in the interval $[0, 4]$ because $f(t)$ is equal to zero at the endpoints of our interval $[0, 4]$. Now the x or, in our case, the t coordinate of the vertex of a parabola of the form $At^2 + Bt + C$ is given by $-\frac{B}{2A}$.

For our function $f(t) = 10t - \frac{5t^2}{2}$, $A = -\frac{5}{2}$, so $2A = -5$. Now, $B = 10$ so the vertex of our parabola has t coordinate equal to $-(\frac{10}{-5}) = 2$. So the high point occurs at $t = 2$ or 10:00 A.M..

3. Of course you can use the methods of calculus to find the value of t for which $f(t)$ attains an absolute maximum on the interval $[0, 4]$. To do this we would take the first derivative of $f(t)$ which would give us $f'(t) = 10 - 5t$ which has a critical number at $t = 2$. Now the value of $f(t)$ at each of the endpoints of the interval $[0, 4]$ is zero. The value of $f(t)$ at $t = 2$ is 10. So again, the absolute maximum of $f(t)$ occurs a $t = 2$, or 10:00A.M..