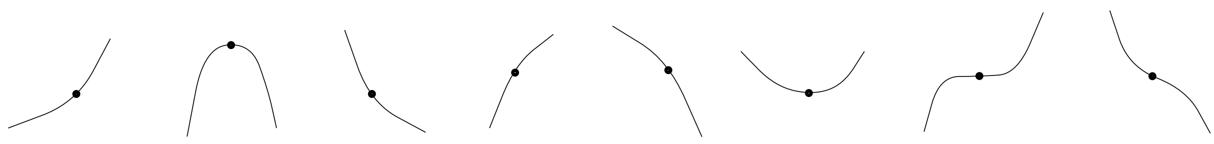
FINAL PRACTICE PROBLEMS FOR MA135 FALL 2007

x	$Q(x)$	$Q'(x)$	$Q''(x)$
0	1	-4	-3
1	2	0	7
2	0	-5	2
3	1	0	-2
4	5	5	-2
5	8	-1	0
6	-2	2	3
7	-2	0	0

1. Suppose Q and its first derivative Q' and its second derivative Q'' have the values indicated in the accompanying table.

Below are some disconnected pieces of the graph of Q. Each value of x matches exactly one picture. Find the matches.



2. The line $y = 3x + 7$ is tangent to the graph of $y = f(x)$ at $x = 4$. What is $f(4)$? What is $f'(4)$.

3. Suppose y is defined implicitly as a function of x by $x^2 + Axy^2 + By^3 = 1$ where A and B are constants to be determined. Given that this curve passes through the point $(3, 2)$ and that its tangent at this point has slope -1, find A and B.

4. Evaluate these indefinite integrals.

a) $\int 7x^2 - 3e^x + \frac{5}{x}\,dx$ b) $\int (x^3+5)^2\,dx$ c) $\int 5\sin x + \cos(5x)\,dx$ d) $\int \frac{x}{x^2+5}\,dx$

5. Evaluate these definite integrals using methods of calculus.

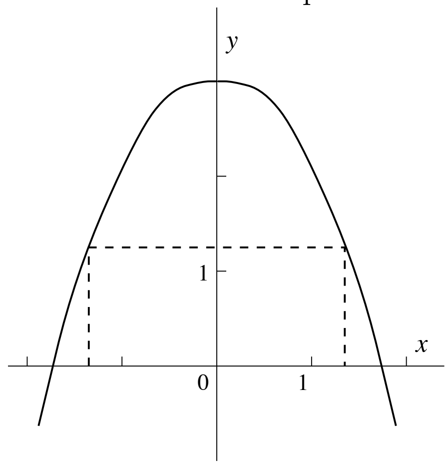
a) $\int_1^2 \left(x^3 - \frac{1}{x^4}\right) dx$ b) $\int_0^{\ln 3} 4e^{2x}\,dx$ c) $\int_0^2 x^2\sqrt{1+3x^3}\,dx$ d) $\int_{\ln \pi}^{\ln 2\pi} e^x \sin(e^x)\,dx$

e) $\int_e^{e^e} \frac{1}{x \ln x}\,dx$

6. Find $\frac{dy}{dx}$.

a. $y = x^{(x^2)}$ b. $y = \frac{3x+5}{7x^2+1}$ c. $y = \ln(3x+5)$ d. $xe^y = \cos(xy)$

7. A rectangle is inscribed as shown in the parabola $y = 3 - x^2$. Of all such rectangles, find the dimensions of the one whose area is maximum. Explain briefly in terms of calculus why



your answer gives a maximum.

8. Use differentials to approximate $\sqrt{103}$.

9. Let $f(3) = 1,\ g(2) = 3,\ f'(3) = 4,\ g'(2) = 5$. If $h(x) = f(g(x))$, find $h'(2)$.

10. Consider the function

$$f(x) = \begin{cases} 9 - 4x & \text{if } x < 1 \\ -x^2 + 6x & \text{if } x \geq 1 \end{cases}$$

on $[0, 4]$.

a. Explain why this function is guaranteed to have an absolute max and an absolute min on $[0,\ 4]$.
b. Find the absolute max and absolute min on $[0,\ 4]$.

11. a. Find $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{2x}\right)^{3x}$. b. Find $\displaystyle\lim_{x\to 0}\left(1+\frac{1}{2x}\right)^{3x}$.

12. Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sqrt{x^2+4}}{x-3}.$$

13. An apartment complex has 90 units. When the monthly rent for each unit is \$600, all units are occupied. Experience indicates that for each \$20-per-month increase in rent, 3

units will become vacant. Each rented apartment costs the owners of the complex $230 to maintain. What monthly rent should be charged to maximize the owners' profits?

14. A manufacturer of light bulbs estimates that the fraction $F(t)$ of bulbs that remain burning after t weeks is given by $F(t) = e^{-kt}$ where k is a positive constant. Suppose twice as many bulbs are burning after 5 weeks as after 9 weeks.

a. Find k and determine the fraction of bulbs still burning after 7 weeks.
b. What fraction of the bulbs burn out before 10 weeks?
c. What fraction of the bulbs can be expected to burn out between the 4th and 5th weeks?

15. A particle is traveling along a straight line, and its distance from the origin is given by the equation

$$s(t) = t^3 - 12t^2 + 36t + 4$$

where $t \geq 0$.

a. What is the average velocity of the particle on the interval [3, 6]?
b. Find c in [3, 6] so the velocity at c is equal to the average velocity you found in the previous question.
c. When is the particle's acceleration positive? Negative?
d. When is the particle speeding up? Slowing down?

16. Consider the function

$$f(x) = \sin x - \cos x$$

on the interval $[0, 2\pi]$.

a. When is f increasing? Decreasing?
b. When is f concave up? Concave down?
c. What are the relative and absolute max/min of f and what are their locations?
d. What are the inflection points of f?
e. Sketch a graph of f.

17. It is easily verified that $x = 2$ and $x = 4$ are both solutions to the equation

$$x^2 = 2^x$$

Show also that there is another solution in the interval $[-1, 1]$. *Hint:* Do not attempt to find an exact solution! You are only asked to show that another solution exists in the specified interval.

18. Approximate $\int_1^2 x^2\, dx$ using a Riemann Sum with 4 sub-intervals using the following methods:

a. Left endpoint.

b. Right endpoint.
c. Midpoint.

19. Suppose that the total cost (in dollars) of manufacturing x units of a certain commodity is

$$C(x) = 4x^2 + 10x + 324$$

at what level of production is the average cost per unit the smallest?

20. Suppose that the total cost (in dollars) of manufacturing x units of a certain commodity is

$$C(x) = \frac{1}{4}x^4 - \frac{43}{3}x^3 + \frac{440}{2}x^2 + 484x + 100$$

at what level of production is the marginal cost $C'(x)$ per unit the smallest?

(For a real exam problem, I'd need to make the numbers work out so that the students wouldn't need calculators. The main point is to recognize that $C''(x) = 0$ needs to be solved, not $C'(x) = 0$. This problem is a stand-in for a variety of such problems.)