

Name _____ MA135 Final Exam A December 16, 2013

Instructor _____ Section _____

Be sure to show all of your work. All solutions should use calculus techniques from this course. Unsupported answers will receive no credit! Calculators are not allowed on this exam. You may only use the formula sheet and scratch paper supplied with this exam. Good Luck!!

Prob No.	Max Pts	Points	Prob No.	Max Pts	Points
1	18		8	17	
2	18		9	18	
3	18		10	18	
4	18		11	18	
5	17		12	18	
6	18		13	18	
7	18		14	18	
Subtotal	125		Subtotal	125	

Grand Total	
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2

1. (9 points each) You do not need to simplify your answers.

a. Find $\lim_{t \rightarrow 1} \frac{t^2 - 1}{t - 1}$.

b. Find $\lim_{x \rightarrow 0} \frac{x^2}{e^{3x} - 1 - 3x}$.

2. A radioactive frog hops out of a pool of radioactive goo and into a lab. The scientists discover that the frog registers 10 Curies of radioactivity initially and that two days later it registers 3 Curies of radioactivity. This is an exponential decay problem.

- a. (12 points) Find a formula for the level, $G(t)$, of the frog's radioactivity after t days.
- b. (6 points) When will its level of radioactivity reach 1 Curie?

$G(t)=$	
Reaches 1 Curie for $t=$	

3. (9 points each) You do not need to simplify your answers.

a. Find $\frac{dy}{dx}$ if $y = \frac{x+1}{\sin x}$.

b. Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x} \left(x + \frac{1}{x} \right)$.

4.

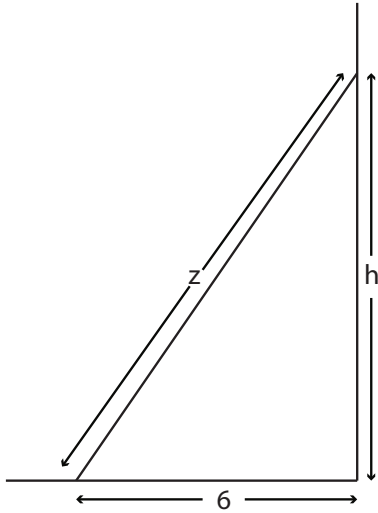
a. (10 points) If $x^3 + 3x^2y - 5y^2 = 12$, find $\frac{dy}{dx}$ at $(2, 2)$.

At $(2,2)$, $\frac{dy}{dx} =$	
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b. (8 points) If $y = x^{2x}$, find $\frac{dy}{dx}$ as a function of x .

$\frac{dy}{dx} =$	
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5. (17 points) The base of an extension ladder is 6 feet from a building. If the length of the ladder is increasing at 2 ft/sec, how fast is the distance from the top of the ladder to the ground increasing when the ladder is 10 feet long? (An extension ladder is a ladder like the one on a fire truck that can be made longer or shorter as the need arises.)



6. (18 points) Let $f(x) = \sqrt[3]{2x+2}$. Use linear approximation or differentials to approximate $f(3.03)$. You do not need to simplify your answer.

7. (18 points) The wicked queen is selling apples to her wicked friends at a price of \$7 per apple and at this price her friends have been buying 45 apples per month. She wishes to raise the price and estimates that for each \$1 increase in price, 3 fewer apples will be sold each month. At which price should she sell the apples so as to maximize her wicked revenue?

8. (17 points) Find the x -values of the absolute maxima and minima of the function $f(x) = \frac{9}{x} + 4x - 10$ on the interval $[1, 3]$. (You must use calculus methods from this course to solve this problem. Plugging in numbers and guessing will receive no credit.)

Absolute max at $x =$	
Absolute min at $x =$	

9. (18 pts) Find the intervals where the function $y = \frac{1}{x} - \frac{1}{x^2}$ is increasing and decreasing, concave up, and concave down. Find all horizontal, vertical asymptotes, relative extrema, and inflections. Write “none” in the blank if there are none. (Hint: $y' = \frac{2-x}{x^3}$ and $y'' = \frac{2x-6}{x^4}$.)

Increasing	
Decreasing	
Concave up	
Concave down	
Horizontal asymptotes	
Vertical asymptotes	
x-values of relative maxima	
x-values of relative minima	
x-values of inflections	

10. (9 points each)

- a. Suppose that the total revenue (in dollars) from sales of x units of a certain commodity is

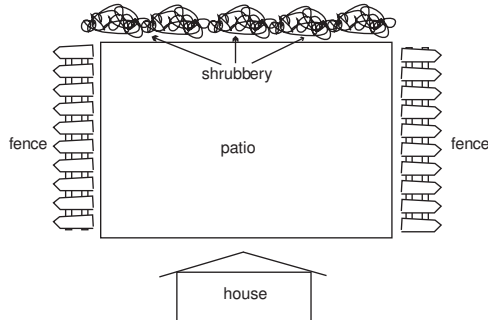
$$R(x) = -2x^2 + 68x - 18, \quad 0 < x < 16$$

At what level of sales is the marginal revenue per unit equal to the average revenue? (Hint: Recall the formula for average cost. The formula for average revenue is similar.)

$x =$	
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- b. Find the area under the curve $y = \frac{1}{\sqrt{x}}$ from $x = 3$ to $x = 5$.

11. (18 points) A contractor plans to build a rectangular patio, with area equal to 800 square feet, behind his house. Fencing and shrubbery (bushes) will be used to give privacy. The fencing material for the sides costs \$25 per foot and the shrubbery along the back costs \$16 per foot. Find the best dimensions assuming that the contractor is very concerned about cost. (There is nothing between the house and the patio.)



fence side=	
shrubby side=	

12. (18 points) Find $\int \frac{x^3 + 3x + 1}{\sqrt{x}} dx$.

13. (18 points) Find $\int \frac{\cos x}{1 + \sin x} dx$.

14. (9 points each)

a. Find $\int_1^2 (3x + 1)^8 dx$.

$\int_1^2 (3x + 1)^8 dx =$	
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You do not need to simplify this answer!

b. If $F(x) = \int_1^{x^3} \frac{1}{1+t^3} dt$, find $F'(x)$.

$F'(x) =$	
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