Review Problems for the first exam in Math 135 Spring 2009.
NOTE : These are only practice problems!

The exam will cover all the material through section 3.5.
1. Find the equation of the line that passes through $(2, 4)$ and is perpendicular to the line $2x + 3y = 12$.

2. Let $f(x) = \sqrt{x^2 + 2x - 15}$, $g(x) = \dfrac{1}{x}$
a) Find the domain of the function $f(x)$.
b) Find $g(f(x))$ and $f(g(x))$.
c) Find the domains of $g(f(x))$.

3. Find the **exact** value of each of the following limits. Show all work and/or give reasons for your answers:

a) $\lim\limits_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4}$

b) $\lim\limits_{x \to 4} \dfrac{x^2 - 5x + 6}{x^2 - 4}$

c) $\lim\limits_{x \to 0} \dfrac{\tan 2x}{\tan 7x}$

d) $\lim\limits_{x \to 0} \dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$

e) $\lim\limits_{x \to 3} \dfrac{|x - 3|}{x - 3}$

f) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$

4. Let $g(x) = x^5 + x^3 + 30$. **Without graphing** the function g, use a theorem to show that there is at least one number $c \in (-2, \; 2)$ such that $g(c) = 0$. HINT: don't try to find c !

5. Let

$$f(x) = \begin{cases} x^2 + 1 & x > 2 \\ A & x = 2 \\ 2x + 1 & 2 > x \geq 0 \\ x^2 + 3 & x < 0 \end{cases}$$

a)For what value of A is f continuous at $x = 2$? **Explain!**
b)Find the following limits or write DNE if the limit doesn't exist. **Show all work.**
$\lim_{x \to 2} f(x)$, $\lim_{x \to 1} f(x)$, $\lim_{x \to 0} f(x)$, $\lim_{x \to (-1)} f(x)$
b) Is $f(x)$ differentiable at $x = 0$?

6. Find the following derivatives from the definition:
a) $f(x) = x^2 + 3x$

b) $g(x) = \dfrac{1}{x + 2}$

c) $h(x) = \sqrt{x - 3}$

7. The line $y = 2x + 3$ is tangent to the parabola $y = x^2 + B$. Find B.

8. Find the derivative of the following functions. Don't simplify!

a) $f(x) = \dfrac{7}{x^{3/7}} + \sqrt{x^5} + x^7 + 45$

b) $g(x) = (x+9) \cdot (x^2 - 7x)$

c) $h(x) = \left(\dfrac{x^2 + 7}{x^5 - 8x}\right)^9$

d) $k(x) = \dfrac{(x^4 + 2)^6}{\sqrt[3]{x^3 + 5x}}$

e) $\ln x^5 + x - \ln x$

9. Find the equation of the tangent line for the graph of $f(x) = 2\sqrt{x} + x^2 - 5$ at $x = 1$.

10. Let $f(x) = g(\sqrt{x + 3})$. Find $f(6)$ and $f'(6)$.
It is impossible to find g, but it will be useful to use some of the following known values of $g(x)$ and $g'(x)$:
$g(1) = 2$, $g(2) = 5$, $g(3) = 7$, $g(4) = 2$, $g(5) = 11$, $g(6) = 13$ and $g(7) = 21$
$g'(1) = 3$, $g'(2) = 2$, $g'(3) = 8$, $g'(4) = 10$, $g'(5) = 12$, $g'(6) = 21$ and $g(7) = 23$.

11. Sketch a possible graph of F on $[-3,\ 3]$ such that:
F is continuous on $[-3,\ 0)$ and $(0,\ 3]$, $\lim_{x \to 0^+} F(x) = 5$, $\lim_{x \to 0^-} F(x) = -2$, F is not differentiable only at $x = 0$ and $x = 1$.

12. Let $y = 2x^4 + 3x^2 + 12$. Find $\dfrac{d^3 y}{dx^3}$.

13. The distance s (in feet) covered by a car t seconds after starting from rest is given by $s(t) = 20t + 6t^2 + t^3$, when $0 \le t \le 20$.
a) What is the velocity of the car 5 seconds after starting from rest?
b) What is the acceleration of the car at that time?

14. Sketch a possible function on the domain $(-2,\ 4)$ that is :
Not differentiable only at $x = (-1.5)$, (-1) , 0 , 0.5 , 1 , 3, not continuous only at $x = (-1)$, 0.5 , 3 and has no limit only at $x = 0.5$, 3.

15. Let $h(x) = f(g(x))$. Assume that $f(1) = 2$, $f'(1) = 7$ $f(2) = 5$ $f'(2) = 5$, $g(1) = 2$ and $g'(1) = 3$. Find $h'(1)$, $(fg)'(1)$, $(f/g)(1)$.

16. Suppose that f and g are differentiable functions such that $f(g(x)) = 8x^2$ to all real numbers x. Assume that $f(2) = 7,\quad g(2) = 4,\quad f'(2) = 4 \quad and \quad f'(4) = 2$. What is $g'(2)$?

17. Give an exact solution for the following population growth problem: The number of students taking 135 increased from 2,300 to 3,200 in the last 2 years. Assuming that this is an exponential growth, find K, the growth constant and use it to find the number of 135 students 3 years from now.

18. Expand the following expression: $\ln \dfrac{\sqrt[5]{x}x^3y^2}{6\sqrt{y}x^9}$

19. Prove that the equation $\cos x - \sin x = x$ has at least one solution on the interval $(0, \frac{\pi}{2})$.

20. The graph of a function $f(x)$ is given below
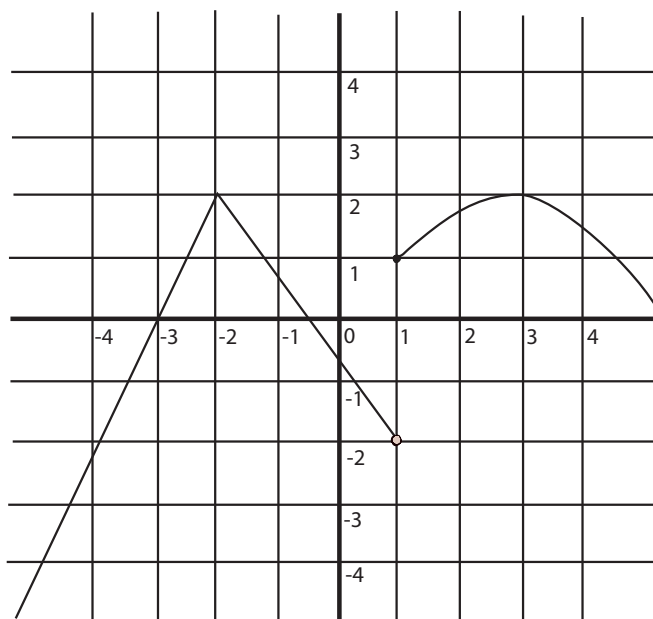
Find all values of x where f fails to be
    a. continuous.
    b. differentiable.
    c. For which values of x is the derivative of f equal to 0?