

Suggestions for Studying for the Second Hour Exam

The following notes are intended as a guide for studying for the second hour exam in Math 135. As was the case with the first hour exam, the exact topics to be covered on the test vary from section to section. Students should check with their lecturer.

The notes for the first hour exam contain many general suggestions that apply to the second hour exam as well. Students should review those notes.

The new skills that may be tested on the second hour exam include the following:

- Compute the first derivative of a function, and perhaps higher derivatives as well, when the function is defined implicitly by an equation in two variables.
- Use the linear approximation (differential) of a function at a point to estimate the value of the function at nearby points.
- Determine the absolute extrema and the location of any relative extrema of a given function.
- Use information about the first and second derivatives of a function to describe important aspects of the shape of the graph of a given function.
- Determine the horizontal and vertical asymptotes, if any, of a given function.
- Recognize indeterminate forms and use l'Hôpital's Rule to evaluate them.

Students frequently say that "word problems" give them trouble. The second hour exam is likely to have two types of word problems, related rate problems and problems that ask for a certain physical or geometric quantity to be optimized. These problems are difficult for several reasons. First, the statements of the problem frequently contain descriptions of physical systems or geometric objects. These descriptions must be understood and sketches produced before the solutions can be started. Second, the solutions often require the use of facts from high school geometry. Most common are the properties of similar figures and formulas for the areas and volumes of basic geometric figures. Students should practice by doing as many related rate and physical optimization problems as they can.

Considerable time has been spent learning how to analyze the shape of the graph of a function by looking at its first and second derivatives. Before graphing calculators became so common, the standard way of testing whether students had learned this material was to ask them to sketch the graph of a given function, making sure that the picture conveyed accurately information about relative extrema, points of inflection, and asymptotes.

Now that graphing calculators can easily produce such pictures, other methods of testing must be found. Students can be asked to compute asymptotes, relative extrema, and points of inflection. As always, work must be shown in answering such questions. Students can also be given the graph of a function and asked to identify its key features. Finally, students can be asked to make up the graph of a function that satisfies certain conditions. Here is an example of a problem of this third type:

Sketch the graph of a function f that satisfies the following conditions:

- $f(x)$ is defined for all x except $x = -1$, $x = 0$, and $x = 1$.
- f has vertical asymptotes at $x = -1$, $x = 0$, and $x = 1$.

- The value of $f(x)$ is never 0.
- The lines $y = -1$ and $y = 1$ are each horizontal asymptotes for f .

The only result discussed since the first hour exam that is deemed important enough to be given a title is the Mean Value Theorem. Students should be sure that they understand the statement of this theorem as well as its direct consequences, such as the fact that if two functions are defined on an interval and have the same derivative, then those functions differ by a constant.