Math. 138 Review Problems, S2008

1. Evaluate the following integrals.

(a) $\displaystyle\int_e^{10} x^6 \ln x dx$ (b) $\displaystyle\int \cos x e^x dx$ (c) $\displaystyle\int \frac{x+2}{x^2-9} dx$

(d) $\displaystyle\int \frac{(x^2)}{(x-1)^{21}} dx$ (e) $\displaystyle\int_1^3 (x+1)\sqrt{x^2+2x-1} dx$

2. Let $I = \displaystyle\int_1^3 e^{x^2} dx$

(a) Use the Trapezoidal and the Simpson's Methods with $n = 4$ to approximate I .

(b) Estimate the error for the Trapezoidal Methods used in (a).

3. Check if each one of the following improper integrals divergent or convergent. Show all work: evaluate the integral and take the appropriate limits.

(a) $\displaystyle\int_0^{\infty} \frac{\ln x}{x} dx$ (b) $\displaystyle\int_1^2 \frac{1}{x-2} dx$ (c) $\displaystyle\int_0^{\infty} xe^{-x} dx$ (d) $\displaystyle\int_1^2 \frac{1}{(x-2)^3} dx$

4. Find the nth Taylor polynomial of each one of the following function about a. In each case estimate the reminder for the given x, $|R_n(x)|$.

(a) $f(x) = e^{2x}$ $n = 4$ $a = 1$ $x = 1.2$

(b) $g(x) = \sin(x/2)$ $n = 6$ $a = 0$ $x = 0.01$

(c) $h(x) = x^2 e^x$ $n = 3$ $a = 0$ $x = 0.5$

5. Solve each of the following differential equations.

(a) $\dfrac{dy}{dx} = 1 + x - y^2 - xy^2$ $y(0) = 4$

(b) $\dfrac{dy}{dx} = e^{2x} + y - 1$ $y(0) = 5$

(c) $y'' - 6y' + 9y = 0$ $y(0) = 3,\ y'(0) = 10$

(d) $x\dfrac{dy}{dx} + y = xe^{2x}$ $x = 1, y = 1$

(e) $y'''' + 4y'' - 5 = 0$

6. Use the method of undetermined coefficient and variation of variable to find the general solution of each one of the following non homogenous second order differential equation $y'' - 3y' - 4y = F(x)$ where $F(x)$ is:

(a) $2\sin x$ (b) $4x^2$ (c) e^x

7. Let $A = \begin{pmatrix} -2 & 2 & 1 \\ 1 & 5 & 1 \\ 6 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 4 \\ 1 & 5 & -7 \end{pmatrix}$

(a) Find the transpose of A. (b) Find the determinant of A.

(c) Find the inverse of A. (d) Find A^2. (e) Evaluate the matrix AB.

(f) Evaluate the matrix $(2B - A)B$.

8. Find the eigenvalues and the eigenvectors of each one of the given matrices.

$$A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 1 & -3 \\ 0 & 0 & 2 \\ 0 & 0 & -3 \end{pmatrix}$$

9. Solve each of the systems of equations using:

(a) Elementary row operation on the augmented matrix of the system.

(b) Inverse matrix.

(c) Cramer's rule .

$$\text{(i)} \begin{array}{rcrcrcr} x & + & y & - & z & = & 1 \\ 3x & - & 2y & + & z & = & 3 \\ 4x & + & y & - & 2z & = & 0 \end{array} \qquad \text{(ii)} \begin{array}{rcrcrcr} x & - & 5y & + & 2z & = & -5 \\ 3x & - & 14y & + & 3z & = & -8 \\ 4x & - & 18y & + & 3z & = & -8 \end{array}$$

10. Real life problems

(a) A famous soccer player signed a contract for 5 years with a new team. He has to chose one of the two options: to get a lump sum of 40 million dollars to his account or to get an income stream of 9 millions dollars per year for the 5 years to the same account. If the account gives an annual continuous compound interest of 3%, which option is better? Does the choice really depend on the annual interest?

(c) The equilibrium population (also called the carrying capacity) of some animal is 1,000 individuals. Its rate of growth without restrictions is 0.05 per year.

(i) Find the differential equation that describes the rate of growth of this population.

(ii) Solve the differential equation you got tin (i).

(iii) The population 4 years ago was estimated to be 300 individuals. What is the number of individuals today?

(d) The population density of Hope City is $D(r) = 100r^2 - 5r + 250$ where r is the distance from the center of the city. How many people are living between 2 to 3 miles form the center of the city?

(e) The number of cells in a growing dish doubles every 3 hours.
(i) Write the differential equation that describes the rate of growth and solve it.
(ii) How many cells the are after 10 hours?
(iii) After how many hours the number of cells will multiply by 7?

(f) How much money you will have in your account at the end of 5 years if it continuously flows in at a rate of $500 + e^{0.5t}$ per year. Assume that you account gives a continuous compounded interest rate of 3%.

(g) A population $P(t)$,satisfying the logistics equation values $P(1970) = 150, \quad P(1980) = 200 \quad and \quad P(1990) = 220$ in thousands of individuals. Find the equation for the population, $P(t)$, at any time t.