Math 151, Fall 2016, Review Problems for Exam 1

These problems are presented in order to help you understand the material that is listed prior to the first exam in the syllabus. DO NOT assume that your first midterm exam will resemble this set of problems. The following 20 problems are not meant to be a sample exam. These problems are just a study aid.

(1) Describe the set $S = \{x \in \mathbf{R} : |2x-4| > 2 \text{ and } |x-3| \le 1\}$ in terms of intervals.

(2) Assume that $f(x)$ is a function with domain $\mathbf{R}$, and that $f(x)$ is increasing on $[5,\infty)$. Explain why (a) and (b) must be true:
(a) If $f(x)$ is an odd function then $f(x)$ is increasing on $(-\infty,-5]$.
(b) If $f(x)$ is an even function then $f(x)$ is decreasing on $(-\infty,-5]$.

(3) Complete the square for $2x^2-8x-10$. Use your answer to find the minimum of $2x^2-8x-10$ and to solve $2x^2-8x-10=0$.

(4) Find functions $f(x)$ and $g(x)$ with domain $\mathbf{R}$ such that $f\circ g \ne g \circ f$.

(5) Find all solutions of $2\sin^2 x = 1+\cos(2x)$ in the interval $[0,2\pi]$.

(6) Simplify $\sin^{-1}(\sin(9\pi/4))$, $\sec(\sin^{-1} x)$ and $\cos(\tan^{-1} x)$.

(7) Solve $\ln(x^2+7)-\ln(x^2+1)=2\ln 2$.

(8) The position of a particle at time t (in seconds) is given by $\dfrac{t}{1+t^2}$ (in feet). Find the average velocity of the particle over the time interval $[1,3]$.

(9) Find the exact values of the following limits. Do not use a calculator. Do not use L'Hôpital's Rule, which appears much later in the textbook.

$$\lim_{x\to 0}\frac{x}{\sin(7x)} \qquad \lim_{x\to 0}\frac{\sin(5x)}{\sin(7x)} \qquad \lim_{x\to 0}\frac{x}{\tan x} \qquad \lim_{x\to 0} x\cos(x^{-3})$$

$$\lim_{x\to 5^+}\frac{x-5}{|x-5|} \qquad \lim_{x\to 5^-}\frac{x-5}{|x-5|} \qquad \lim_{x\to 3^+}\frac{x^2-20}{x^2-9} \qquad \lim_{x\to 3^-}\frac{x^2-20}{x^2-9}$$

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2+2x-8} \qquad \lim_{x\to 2}\frac{x^3-2x^2+x-2}{x^3-x^2-x-2}$$

$$\lim_{x\to 3}\frac{4-\sqrt{5x+1}}{5-\sqrt{8x+1}} \qquad \lim_{x\to 3}\frac{4-\sqrt{5x+1}}{6-2x} \qquad \lim_{x\to 0}\frac{1-\sec x}{x^2}$$

(10) Find constants a, b, c such that the function $f(x)$, defined below, is continuous.

$$f(x) = \begin{cases} ax^2+b & \text{if } x \le -1, \\ bx+c & \text{if } -1 < x < 1, \\ 2c & \text{if } x = 1, \\ \frac{8}{1+x^2} & \text{if } 1 < x. \end{cases}$$

There are more problems on the next page.

(11) Explain why $x = \cos x$ must have a solution.

(12) Use the ε, δ definition of limit to prove $\lim_{x\to 2} 3x + 4 = 10$.

(13) Assume $f(x) = x^{-2}$. Find $f'(x)$ using the limit definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

of the derivative.

(14) A batter hits a pitched baseball. The height of the baseball is $-16t^2 + 12t + 4$ feet at time t seconds after the bat meets the ball. An outfielder catches the ball when his glove is 6 feet above the ground. At what time did the fielder grab the baseball? What was the maximum height of the ball?

(15) A function $f(x)$ is defined by

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < 1, \\ 3x + 2 & \text{if } x \geq 1. \end{cases}$$

show that this function is continuous, but not differentiable.

(16) Find constants a, b such that the function $f(x)$, defined below, is differentiable.

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < 2, \\ ax^2 + b & \text{if } x \geq 2. \end{cases}$$

(17) Find the derivatives of the following functions of x:

$$(x^3 + x)^5(1 + \cos x)^9 \qquad \frac{\cot x}{1 + e^{4x}} \qquad \sin\left(\sqrt{x^4 + x^2 + 3}\right) \qquad \csc(e^x + \sqrt{x})$$

(18) Find the second derivatives of the following functions of x:

$$(3 + x^{-3})^5 \qquad \tan(7x) \qquad \frac{1}{\sqrt{e^x + \cos x}} \qquad e^{x^2+4x+3}$$

(19) Find the first, second, third and fourth derivatives of $y = \cos(2x)$.

(20) Assume $f(x) = e^{-x^2}$ and $g(x) = \dfrac{1}{1 + x^2}$. Solve $f''(x) = 0$ and $g''(x) = 0$.