Math 151, Spring 2012, Review Problems for Exam 1

Your first exam is likely to have problems that do not resemble these review problems.

(1) Describe the set $S = \{x \in \mathbf{R} : |2x-4| > 2 \text{ and } |x-3| \le 1\}$ in terms of intervals.

(2) Assume that $f(x)$ is a function with domain $\mathbf{R}$, and that $f(x)$ is increasing on $[5,\infty)$. Explain why (a) and (b) must be true:
(a) If $f(x)$ is an odd function then $f(x)$ is increasing on $(-\infty,-5]$.
(b) If $f(x)$ is an even function then $f(x)$ is decreasing on $(-\infty,-5]$.

(3) Complete the square for $2x^2-8x-10$. Use your answer to find the minimum of $2x^2-8x-10$ and to solve $2x^2-8x-10=0$.

(4) Find functions $f(x)$ and $g(x)$ with domain $\mathbf{R}$ such that $f\circ g \neq g\circ f$.

(5) Find all solutions of $2\sin^2 x = 1+\cos(2x)$ in the interval $[0,2\pi]$.

(6) Simplify $\sin^{-1}(\sin(9\pi/4))$, $\sec(\sin^{-1} x)$ and $\cos(\tan^{-1} x)$.

(7) Solve $\ln(x^2+7)-\ln(x^2+1) = 2\ln 2$.

(8) The position of a particle at time t (in seconds) is given by $\dfrac{t}{1+t^2}$ (in feet). Find the average velocity of the particle over the time interval $[1,3]$.

(9) Find the exact values of the following limits. Do not use a calculator. Do not use L'Hôpital's Rule, which appears much later in the textbook.

$$\lim_{x\to 0}\frac{x}{\sin(7x)} \qquad \lim_{x\to 0}\frac{\sin(5x)}{\sin(7x)} \qquad \lim_{x\to 0}\frac{x}{\tan x} \qquad \lim_{x\to 0} x\cos(x^{-3})$$

$$\lim_{x\to 5^+}\frac{x-5}{|x-5|} \qquad \lim_{x\to 5^-}\frac{x-5}{|x-5|} \qquad \lim_{x\to 3^+}\frac{x^2-20}{x^2-9} \qquad \lim_{x\to 3^-}\frac{x^2-20}{x^2-9}$$

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2+2x-8} \qquad \lim_{x\to 2}\frac{x^3-2x^2+x-2}{x^3-x^2-x-2}$$

$$\lim_{x\to 3}\frac{4-\sqrt{5x+1}}{5-\sqrt{8x+1}} \qquad \lim_{x\to 3}\frac{4-\sqrt{5x+1}}{6-2x} \qquad \lim_{x\to 0}\frac{1-\sec x}{x^2}$$

(10) Find constants a, b, c such that the function $f(x)$, defined below, is continuous.

$$f(x) = \begin{cases} ax^2+b & \text{if } x \le -1, \\ bx+c & \text{if } -1 < x < 1, \\ 2c & \text{if } x = 1, \\ \frac{8}{1+x^2} & \text{if } 1 < x. \end{cases}$$

There are more problems on the next page.

(11) Explain why $x = \cos x$ must have a solution.

(12) Use the ε, δ definition of limit to prove $\lim_{x \to 2} 3x + 4 = 10$.

(13) Assume $f(x) = x^{-2}$. Find $f'(x)$ using the limit definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

of the derivative.

(14) A batter hits a pitched baseball. The height of the baseball is $-16t^2 + 12t + 4$ feet at time t seconds after the bat meets the ball. An outfielder catches the ball when his glove is 6 feet above the ground. At what time did the fielder grab the baseball? What was the maximum height of the ball?

(15) A function $f(x)$ is defined by

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < 1, \\ 3x + 2 & \text{if } x \geq 1. \end{cases}$$

show that this function is continuous, but not differentiable.

(16) Find constants a, b such that the function $f(x)$, defined below, is differentiable.

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < 2, \\ ax^2 + b & \text{if } x \geq 2. \end{cases}$$

(17) Find the derivatives of the following functions of x:

$$(x^3 + x)^5 (1 + \cos x)^9 \qquad \frac{\tan x}{1 + e^{4x}} \qquad \sin\left(\sqrt{x^4 + x^2 + 3}\right) \qquad \sec(e^x + \sqrt{x})$$

(18) Find the second derivatives of the following functions of x:

$$(3 + x^{-3})^5 \qquad \tan(7x) \qquad \frac{1}{\sqrt{e^x + \cos x}} \qquad e^{x^2 + 4x + 3}$$

(19) Find the first, second, third and fourth derivatives of $y = \cos(2x)$.

(20) Assume $f(x) = e^{-x^2}$ and $g(x) = \dfrac{1}{1 + x^2}$. Solve $f''(x) = 0$ and $g''(x) = 0$.