Math 151, Fall 2009, Review Problems for Exam 2

Your second exam is likely to have problems that do not resemble these review problems. Partial answers to these problems will be posted in a few days.

(1) Find $\displaystyle\lim_{x\to 1} \frac{2x^4 - 3x^3 + x^2 - x + 1}{x^4 - 3x^3 + 2x^2 + x - 1}$.

(2) Find $\displaystyle\lim_{x\to 0} \frac{x - \sin x}{x - x \cos x}$.

(3) Find the horizontal asymptotes of $f(x) = \dfrac{x}{\sqrt{7x^2 + 1}}$.

(4) For each function given below, find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima, the inflection points, the horizontal asymptotes and the vertical asymptotes.

(a) $f(x) = \dfrac{x^2}{x^2 - 4}$ (b) $g(x) = \dfrac{x}{x^2 - 4}$.

(5) For the function $f(x) = x^5 - 3x^3 + 4x$, find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima and the inflection points.

(6) For the function $f(x) = \dfrac{1}{\sqrt{x^2 + 1}}$, find the intervals where it is concave up, the intervals where it is concave down and the inflection points.

(7) Find all local maxima and all local minima of $f(x) = \cos x - \dfrac{\sin x}{\sqrt{3}}$.

(8) Find $\dfrac{d}{dx}\left[\sec^{-1} x\right]$ using the identity $\sec^{-1} x = \cos^{-1}(1/x)$ and the Chain Rule. The algebra must be done carefully to get $|x|$ in the denominator.

(9) Find the absolute maximum and the absolute minimum of $f(x) = \ln(-x) + \sec^{-1} x$ over the interval $[-2, -1]$.

(10) Find $\dfrac{d}{dx}\left[(\ln x)^x\right]$ for $x > 1$.

(11) Find the slope of the tangent to the curve $x^2 y^3 + xy = 78$ at the point $(3, 2)$ on the curve.

(12) Find the linearization of $f(x) = x^{1/3}$ with center $a = 27$.

(13) A spherical weather balloon is being inflated at the rate of 12 cubic inches per second. What is the radius of the balloon when its surface area is increasing at a rate of 5 square inches per second?

(14) Find the area between the x-axis, the line $x = 1$ and the parabola $y = x^2$ in the following way: Approximate the area using the sum of the areas of n rectangles. Let n approach infinity.

(15) Find $\sqrt{7}$ with an accuracy of 0.000001 using Newton's Method.

(16) Find all functions $f(x)$ such that $f''(x) = x(x^2 - 1)$.

(17) A medieval farmer follows the same routine every morning: He carries an empty bucket from his house to a river, he fills the bucket with river water, and then he takes it to the goat he keeps in an enclosure. Since famines are a fact of life in the Middle Ages, the farmer is keenly aware of how much food is required for each type of labor. He knows that carrying an empty bucket burns up one slice of bread per mile. He also knows that he burns up three slices of bread per mile when the bucket is filled with water for his goat. The farmer often wonders whether he can save on bread consumption by choosing carefully the exact spot on the river where he gets the water. Since the invention of calculus is still centuries away, the farmer can't figure out how to determine this spot. He does not know that his descendants will use calculus to obtain the answer as outlined in (a) - (f). If the farmer had some spare time to think, he might have solved the problem graphically using a map of his vicinity, a string and two toothpicks. This graphical solution will be explained when the partial answers are posted.

(a) We will use miles as units of measure. Assume that this farmer's house is located at $(1, 1)$, the goat enclosure is at $(-1, \sqrt{3})$, and the river runs along the x-axis. Find a formula for the function $f(x)$ that represents the amount of bread (measured in slices) required for the farmer's morning routine, where $(x, 0)$ is the point on the river where the farmer fills the bucket.

(b) Find numbers a, b, c, d such that every critical point of $f(x)$ is a solution of the equation
$$ax^4 + bx^2 + cx + d = 0.$$

(c) Show that $-1/2$ is a solution of $ax^4 + bx^2 + cx + d = 0$.

(d) Show that $-1/2$ is a critical point of $f(x)$. This is not a consequence of (c). There is a solution of $ax^4 + bx^2 + cx + d = 0$ which is not a critical point of $f(x)$.

(e) Show that $f'(x)$ is a strictly increasing function.

(f) Conclude that $x = -1/2$ minimizes the bread consumption.