Math 151, Fall 2010, Review Problems for Exam 2

Your second exam is likely to have problems that do not resemble these review problems. Partial answers to these problems will be posted in a few days.

(1) Differentiate the following functions of x:

(a) $f(x) = 10^{\arccos x}$ (b) $g(x) = \sin^{-1}(\log_5(x^3 + x + 1))$ (c) $h(x) = \ln(\tan^{-1} x)$.

(2) Find (a) $\displaystyle\lim_{x \to 1} \frac{2x^4 - 3x^3 + x^2 - x + 1}{x^4 - 3x^3 + 2x^2 + x - 1}$ and (b) $\displaystyle\lim_{x \to 0} \frac{x - \sin x}{x - x\cos x}$.

(3) Find (a) $\displaystyle\lim_{x \to 0^+} x^{1/10} \ln x$ and (b) $\displaystyle\lim_{x \to \infty} \frac{\ln x}{x^{1/10}}$.

(4) Find the horizontal asymptotes of $f(x) = \dfrac{x}{\sqrt{7x^2 + 1}}$.

(5) For each function given below, find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima, the inflection points, the horizontal asymptotes and the vertical asymptotes.

(a) $f(x) = \dfrac{x^2}{x^2 - 4}$ (b) $g(x) = \dfrac{x}{x^2 - 4}$.

(6) For the function $f(x) = x^5 - 3x^3 + 4x$, find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima and the inflection points.

(7) For the function $f(x) = \dfrac{1}{\sqrt{x^2 + 1}}$, find the intervals where it is concave up, the intervals where it is concave down and the inflection points.

(8) Find all local maxima and all local minima of $f(x) = \cos x - \dfrac{\sin x}{\sqrt{3}}$.

(9) Find $\dfrac{d}{dx}\left[\sec^{-1} x\right]$ using the identity $\sec^{-1} x = \cos^{-1}(1/x)$ and the Chain Rule. The algebra must be done carefully to get $|x|$ in the denominator.

(10) Find the absolute maximum and the absolute minimum of $f(x) = \ln(-x) + \sec^{-1} x$ over the interval $[-2, -1]$.

(11) Find $\dfrac{d}{dx}\left[(\ln x)^x\right]$ for $x > 1$.

(12) Find the slope of the tangent to the curve $x^2 y^3 + xy = 78$ at the point $(3, 2)$ on the curve.

(13) Find the linearization of $f(x) = x^{1/3}$ with center $a = 27$. Is this linearization greater than or smaller than $f(x)$ when $x \neq 27$?

(14) A spherical helium balloon is given a very small amount of extra helium, so that its radius increases by 0.001 percent. What is the percentage increase in its volume? What is the percentage increase in its surface area?

(15) A spherical weather balloon is being inflated at the rate of 12 cubic inches per second. What is the radius of the balloon when its surface area is increasing at a rate of 5 square inches per second?

(16) Find the area between the x-axis, the line $x = 1$ and the parabola $y = x^2$ in the following way: Approximate the area using the sum of the areas of n rectangles. Let n approach infinity.

(17) Find $\sqrt{7}$ with an accuracy of 0.000001 using Newton's Method.

(18) Find all functions $f(x)$ such that $f''(x) = x(x^2 - 1)$.

(19) Consider the function $f(x) = x^{2/3}(1 - x^2)^2$. Find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima and the inflection points.

(20) Consider the function $f(x) = \dfrac{x}{1 + x^2}$. Find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the absolute maxima, the absolute minima and the inflection points.

(21) A closed box with rectangular sides is built according to the following specifications: The top and bottom sides are made of a material that costs 5 dollars per square foot. the four vertical sides are made of a material that costs 3 dollars per square foot. The top and bottom sides are rectangles with sides of length x and y, where $2x = 7y$. The total cost of materials is 100 dollars. Find the largest possible volume that such a box can hold.

(22) We want to make a conical drinking cup out of paper. It should hold exactly 100 cubic inches of water. Find the dimensions of a cup of this type that minimizes the surface area of the cup.

(23) We want to build a cylindrical can with total surface area of 100 square inches. This surface area includes the top and bottom of the can. Find the dimensions of the can of this type that maximizes the volume of the can.

(24) Consider a differentiable function $f(x)$ with the following properties: $f(4) = 5$ and $f'(x) < -2$ for all x. what can you conclude about $f(7)$? Your answer should be an inequality.