Partial solutions to the Math 151 review problems for Exam 2

These are not complete solutions. They are only intended as a way to check your work.

(1)(a) The derivative is $-\dfrac{(\ln 10)10^{\arccos x}}{\sqrt{1-x^2}}$.

(1)(b) The derivative is $\dfrac{3x^2+1}{(\ln 5)(x^3+x+1)\sqrt{1-[\log_5(x^3+x+1)]^2}}$.

(1)(c) The derivative is $\dfrac{1}{(1+x^2)\tan^{-1}x}$.

(2)(a) The limit is -4. The L'Hôpital Rule (used twice) is much more efficient than using long division to factor $(x-1)^2$ out of the numerator and denominator.

(2)(b) The L'Hôpital Rule (used three times) gives 1/3. In Math 152 you will learn a more efficient way to find this limit.

(3)(a) $\displaystyle\lim_{x\to 0^+} x^{1/10}\ln x = \lim_{x\to 0^+}\frac{\ln x}{x^{-1/10}} = \lim_{x\to 0^+}\frac{1/x}{-(1/10)x^{-11/10}} = \lim_{x\to 0^+} -10x^{1/10} = 0$.

(3)(b) $\displaystyle\lim_{x\to\infty}\frac{\ln x}{x^{1/10}} = \lim_{x\to\infty}\frac{1/x}{(1/10)x^{-9/10}} = \lim_{x\to\infty} 10x^{-1/10} = 0$.

(4) The horizontal asymptotes are $y=\frac{1}{\sqrt{7}}$ and $y=-\frac{1}{\sqrt{7}}$.

(5)(a) We get $f'(x) = -\dfrac{8x}{(x^2-4)^2}$, $f''(x) = \dfrac{24x^2+32}{(x^2-4)^3}$. The function is increasing on $(-\infty,-2)$ and $(-2,0)$. The function is decreasing on $(0,2)$ and $(2,\infty)$. The function is concave up on $(-\infty,-2)$ and $(2,\infty)$. The function is concave down on $(-2,2)$. There is a local maximum at $x=0$. There are no local minima. There are no inflection points. The horizontal asymptote is $y=1$. The vertical asymptotes are $x=\pm 2$.

(5)(b) We get $g'(x) = -\dfrac{x^2+4}{(x^2-4)^2}$, $g''(x) = \dfrac{x(2x^2+24)}{(x^2-4)^3}$. The function is not increasing on any interval. The function is decreasing on $(-\infty,-2)$, $(-2,2)$, $(2,\infty)$. The function is concave up on $(-2,0)$ and $(2,\infty)$. The function is concave down on $(-\infty,-2)$, $(0,2)$. There are no local extrema. There is an inflection point at $x=0$. The horizontal asymptote is $y=0$. The vertical asymptotes are $x=\pm 2$.

(6) The function is increasing on $(-\infty,-1)$, $(-2/\sqrt{5},2/\sqrt{5})$, $(1,\infty)$. The function is decreasing on $(-1,-2/\sqrt{5})$ and $(2/\sqrt{5},1)$. There are local maxima at $x=-1$ and $x=2/\sqrt{5}$. There are local minima at $x=-2/\sqrt{5}$ and $x=1$. The function is concave up

on $(-3/\sqrt{10}, 0)$ and $(3/\sqrt{10}, \infty)$. The function is concave down on $(-\infty, -3/\sqrt{10})$ and $(0, 3/\sqrt{10})$. The inflection points are at $x = 0$ and $x = \pm 3/\sqrt{10}$.

(7) We get $f'(x) = -\dfrac{x}{(x^2+1)^{3/2}}$ and $f''(x) = \dfrac{2x^2-1}{(x^2+1)^{5/2}}$. The function is concave up on $(-\infty, -1/\sqrt{2})$ and $(1/\sqrt{2}, \infty)$. The function is concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$. The inflection points are at $x = \pm 1/\sqrt{2}$.

(8) The local maxima (which are also absolute maxima) occur at $-\pi/6 + 2n\pi$. The local minima (which are also absolute minima) occur at $5\pi/6 + 2n\pi$. The number n represents an arbitrary integer.

(9) Use $x^2 = |x|^2 = |x| \cdot |x| = |x|\sqrt{x^2}$ to obtain the formula as it is usually written.

(10) The absolute minimum is at $x = -\sqrt{2}$. The absolute maximum is at $x = -1$.

(11) The derivative is $(\ln x)^x(\ln(\ln x) + 1/\ln x)$.

(12) The slope of the tangent is $-50/111$. In order to appreciate the usefulness of implicit differentiation, try the following cumbersome alternative method: Solve for x, find the derivative dx/dy, take the reciprocal of this derivative evaluated at $y = 2$.

(13) The linearization is $L(x) = 2 + x/27$. The linearization is greater than $f(x)$ when $x \neq 27$.

(14) The volume increases by approximately 0.003 percent. The surface area increases by approximately 0.002 percent.

(15) If R is the radius, V is the volume and A is the area, then the formulas $V = (4/3)\pi R^3$ and $A = 4\pi R^2$ lead to $\dfrac{12}{5} = \dfrac{dV/dt}{dA/dt} = \dfrac{4\pi R^2(dR/dt)}{8\pi R(dR/dt)} = \dfrac{R}{2}$. We are giving you all of the details of the answer because forgetting the (dR/dt) in the Chain Rule would lead to the same correct answer $R = 24/5$ inches. Getting the right final answer does not guarantee that the method is correct.

(16) We can use thin rectangles of width $1/n$ with right upper vertex on the curve $y = x^2$. The total area of these rectangles is

$$\sum_{j=1}^{n} \frac{1}{n}\left(\frac{j}{n}\right)^2 = \frac{1}{n^3}\sum_{j=1}^{n} j^2 = \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{1}{6} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n}.$$

This approaches the correct area $1/3$ as n approaches infinity.

(17) Use $f(x) = x^2 - 7$ and a positive initial guess. If you use a negative initial guess, then you will approximate $-\sqrt{7}$.

(18) $f(x) = \dfrac{x^5}{20} - \dfrac{x^3}{6} + Cx + D$ for any C and D.

(19) We get

$$f'(x) = (2/3)x^{-1/3}(1 - x^2)(1 - 7x^2)$$

and

$$f''(x) = (2/9)x^{-4/3}(77x^4 - 40x^2 - 1).$$

The function is increasing on $(-1, -1/\sqrt{7})$, $(0, 1/\sqrt{7})$, $(1, \infty)$. The function is decreasing on $(-\infty, -1)$, $(-1/\sqrt{7}, 0)$, $(1/\sqrt{7}, 1)$. There are local maxima at $x = \pm 1/\sqrt{7}$. There are local minima (which are absolute minima) at ± 1 and 0. There are inflection points at

$$x = \pm\sqrt{\frac{20 + \sqrt{477}}{77}}.$$

The function is concave up on

$$\left(-\infty, -\sqrt{\frac{20 + \sqrt{477}}{77}}\right) \quad , \quad \left(\sqrt{\frac{20 + \sqrt{477}}{77}}, \infty\right).$$

The function is concave down on

$$\left(-\sqrt{\frac{20 + \sqrt{477}}{77}}, 0\right) \quad , \quad \left(0, \sqrt{\frac{20 + \sqrt{477}}{77}}\right).$$

(20) We get $f'(x) = \dfrac{1 - x^2}{(1 + x^2)^2}$ and $f''(x) = \dfrac{x(2x^2 - 6)}{(1 + x^2)^3}$. The function is increasing on $(-1, 1)$. The function is decreasing on $(-\infty, -1)$ and $(1, \infty)$. The function is concave up on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$. The function is concave down on $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$. There is an absolute maximum at $x = 1$. There is an absolute minimum at $x = -1$. There are inflection points at $x = 0$ and $x = \pm\sqrt{3}$.

(21) The volume is maximized when $x = \sqrt{35/3}$ feet and $y = (2/7)\sqrt{35/3}$ feet.

(22) The surface area is minimized when the height of the cone is $(600/\pi)^{1/3}$ inches. The surface area is the area of the "pacman" obtained when we make a straight line cut in the cone from the vertex to a point on the base of the cone, and then unravel the cone so that it is completely flat.

(23) The answer should have $H = 2R$, where H is the height of the cylinder, and R is the radius of the top (and bottom) circle.

(24) The inequality $f(7) < -1$ is a consequence of the Mean Value Theorem.