

Partial solutions to the Math 151 review problems for Exam 2

These are not complete solutions. They are only intended as a way to check your work.

(1) The limit is -4 . The L'Hôpital Rule is much more efficient than using long division to factor $(x - 1)^2$ out of the numerator and denominator.

(2) The L'Hôpital Rule gives $1/3$. In Math 152 you will learn a more efficient way to find this limit.

(3) The horizontal asymptotes are $y = \frac{1}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$.

(4) All of your answers except horizontal asymptotes can be checked using your graphing calculator and a viewing window with $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$. The horizontal asymptote for (a) is $y = 1$. The horizontal asymptote for (b) is $y = 0$.

(5) The critical points are $x = \pm 1$ and $x = \pm 2/\sqrt{5}$. The inflection points are $x = 0$ and $x = \pm 3/\sqrt{10}$. If you use your graphing calculator to check the remaining answers, then you will have to choose the graphing window carefully to see confirmation of the following: The function is decreasing on $(-1, -2/\sqrt{5})$ and $(2/\sqrt{5}, 1)$. This is not obvious if the viewing window is $-2 \leq x \leq 2$, $-4 \leq y \leq 4$, for example.

(6) The inflection points are $x = \pm 1/\sqrt{2}$. Your graphing calculator will confirm the remaining correct answers.

(7) The local maxima (which are also absolute maxima) occur at $-\pi/6 + 2n\pi$. The local minima (which are also absolute minima) occur at $5\pi/6 + 2n\pi$. The number n represents an arbitrary integer.

(8) Use $x^2 = |x|^2 = |x| \cdot |x| = |x|\sqrt{x^2}$ to obtain the formula as it is usually written.

(9) The absolute minimum is at $x = -\sqrt{2}$. The absolute maximum is at $x = -1$.

(10) The derivative is $(\ln x)^x(\ln(\ln x) + 1/\ln x)$.

(11) The slope of the tangent is $-50/111$. In order to appreciate the usefulness of implicit differentiation, try the following cumbersome alternative method: Solve for x , find the derivative dx/dy , take the reciprocal of this derivative evaluated at $y = 2$.

(12) The linearization is $L(x) = 2 + x/27$.

(13) If R is the radius, V is the volume and A is the area, then the formulas $V = (4/3)\pi R^3$ and $A = 4\pi R^2$ lead to $\frac{12}{5} = \frac{dV/dt}{dA/dt} = \frac{4\pi R^2(dR/dt)}{8\pi R(dR/dt)} = \frac{R}{2}$. We are giving you all of the details of the answer because forgetting the (dR/dt) in the Chain Rule would lead to the

same correct answer $R = 24/5$ inches. Getting the right final answer does not guarantee that the method is correct.

(14) We can use thin rectangles of width $1/n$ with right upper vertex on the curve $y = x^2$. The total area of these rectangles is

$$\sum_{j=1}^n \frac{1}{n} \left(\frac{j}{n}\right)^2 = \frac{1}{n^3} \sum_{j=1}^n j^2 = \frac{1}{n^3} \left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{1}{6} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n}.$$

This approaches the correct area $1/3$ as n approaches infinity.

(15) Use $f(x) = x^2 - 7$ and a positive initial guess. If you use a negative initial guess, then you will approximate $-\sqrt{7}$.

(16) $f(x) = \frac{x^5}{20} - \frac{x^3}{6} + Cx + D$ for any C and D .

(17) The derivative of $f(x)$ is $A + B$, where $A = \frac{3(x+1)}{\sqrt{(x+1)^2+3}}$ and $B = \frac{(x-1)}{\sqrt{(x-1)^2+1}}$.

If we rewrite $A + B = 0$ as $A = -B$, square both sides and cross-multiply, then we get $8x^4 - 10x^2 + 24x + 14 = 0$. The number $x = -1/2$ is a solution of this equation and a solution of $f'(x) = 0$.

The farmer could have approximated the solution on a map in the following way: For ease of handling, we will use two thin pegs instead of toothpicks. Tie a long string to a peg. Use the peg to pierce the map at the location of the goat enclosure. Once this peg is firmly anchored in the map, hold another peg over the river. Loop the string around the second peg, loop it again around the anchored peg, and loop it yet again around the second peg. Keeping the second peg over the river, pull the free end of the string in the direction of the farmer's house. Holding the second peg in one hand while pulling the string with the other hand, follow the river with the second peg and make sure that the string is taut everywhere and passes over the farmer's house. The farmer's problem is solved when the fingers holding the string extend as far as possible beyond the farmer's house. Then the second peg is over the spot on the river where the bucket should be filled. The string indicates the path that the farmer should take. This method works because there are three string lengths between the two pegs, but only one string length between the second peg and the farmer's house. This corresponds to the ratio of three slices of bread to one slice of bread. A medieval farmer could have invented this method after looking at the ancient pulleys that were used at the time for construction projects in the towns and cities.