

Math 151, Fall 2009, Review Problems for the Final Exam

Your final exam is likely to have problems that do not resemble these review problems. You should also look at the review problems for the first two exams.

(1) Find the largest interval $[a, b]$ such that $\sin x \geq \sqrt{3} \cos x$ for all x in $[a, b]$. Find the area of the region bounded by $y = \sin x$, $y = \sqrt{3} \cos x$ between $x = a$ and $x = b$.

(2) A continuous function $f(x)$ on the interval $[1, 10]$ has the properties $\int_1^8 f(x) dx = 14$, $\int_4^{10} f(x) dx = 7$, $\int_1^{10} f(x) dx = 2$. Find $\int_4^8 f(x) dx$.

(3) Evaluate $\int (1+x)(2+3x) dx$, $\int_{-3}^{-2} \frac{2+3x^2}{x} dx$, $\int_{-1}^2 |x-1| dx$.

(4) Find $\int x^2 e^{x^3+4} dx$, $\int \sin x \cos x dx$, $\int \tan x \sec^2 x dx$.

(5) A bacterial population quadruples in size every 7 days. How many days does it take for this population to triple in size?

(6) Explain why the function $f(x) = \begin{cases} 2x+1 & \text{if } x \leq 1, \\ 4x-1 & \text{if } x > 1 \end{cases}$ is continuous but not differentiable.

(7) A continuous function $f(x)$ is defined by $f(x) = \begin{cases} |x| \ln |x| & \text{if } x \neq 0, \\ a & \text{if } x = 0, \end{cases}$ where a is a constant. Find a . Is $f(x)$ a differentiable function? Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Hint: Look at the case $x \geq 0$ and use symmetry.

(8) Let $f(x)$ be defined for $x > 0$ by $f(x) = x^2 \ln x$. Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

(9) Consider $f(x) = \frac{x^6}{6} - \frac{x^4}{4}$. Find the local maxima, local minima and inflection points. Find the intervals where $f(x)$ is increasing, the intervals where it is decreasing, the intervals where it is concave up and the intervals where it is concave down.

(10) Find $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$, $\lim_{x \rightarrow \infty} \frac{x^3}{e^{x/10}}$, $\lim_{x \rightarrow 0} \frac{1 - \cos(5x)}{1 - \cos(7x)}$, $\lim_{x \rightarrow 0} \frac{\int_0^x \sin(t^2) dt}{x^3}$.

(11) Find the intervals where e^{-x^4} is concave up and the intervals where it is concave down.

(12) Find the absolute maximum of $x^5(1-x)^4$ over the interval $[0, 1]$.

(13) Find the horizontal and vertical asymptotes of $\frac{\sqrt{x^2 + 4x + 7}}{x + 2}$.

(14) Find the points on the curve $y = x^2$ closest to the point $(0, 5)$ in the xy plane.

(15) Differentiate $\tan^{-1}(e^{x^2})$, $\sin^{-1}(\ln x)$, $\sec^{-1}(x^3)$, $\ln(\tan^{-1} x)$, $\cos^2(e^{5x} + x^2)$, x^x .

(16) Find the second derivatives of $(x^2 + 1)^{10}$, $\frac{x}{x^2 + 1}$, $\sin^{-1} x$.

(17) A town boasts a large clock tower. The hour hand of the clock is 12 feet long. The minute hand of the clock is 16 feet long. To save money during tough economic times, the town's council has cut funding for the illumination of the clock face. To compensate, we have a yellow light bulb in the center of the clock and we also have green light bulbs at the tips of the two clock hands. If you face the clock from far away, then it is just barely possible to tell time at night with this minimalistic arrangement. Find the rate of change of the distance between the two green lights with respect to time when the clock strikes 9 pm. Hint: The position of each green light is given by two variables.

(18) Notation: When we say that a picture is of size a by b , the dimension a is the width of the picture. This is the story of a celebrity called A, the pet B of the celebrity, and the webmaster C who maintains A's web page. This web page has a blue rectangle of size 1000 by 1800 pixels. Webmaster C has been given the following job: He has to scan an 8 by 10 photo of A and a 5 by 7 photo of B. These scans must be scaled and put inside the blue rectangle so that the upper left corner of the A picture coincides with the upper left corner of the blue rectangle, the upper right corner of the B picture coincides with the upper right corner of the blue rectangle, the two pictures touch but do not overlap. Scaling means that an a by b picture becomes a c by d picture with $a/b = c/d$. This avoids distortion.

(a) The webmaster must have text in the part of the blue rectangle that remains after the pictures are included. To maximize the writing area, C decides to minimize the sum of the areas of the two pictures. Find the size (in pixels) of the resulting A and B pictures on the web page.

(b) Webmaster C made the mistake of explaining all this on A's reality show. Celebrity A stormed into C's office and yelled "I don't pay you to minimize us. Redo the web page, but this time MAXIMIZE the sum of the areas of the two pictures. To keep you honest, my accountant will sit next to you when you explain how you did it on the next segment of my reality show." Find the size (in pixels) of the resulting A and B pictures when A's instructions are followed.

(c) Webmaster C knew that the answer to (b) would not be acceptable to a person with A's vanity. Fortunately, C found a rare 4 by 8 photo of A. C used this new photo to replace the old 8 by 10 photo. Find the new size (in pixels) of the resulting A and B pictures when A's instructions are followed. Hint: Recall the vertical dimension of the blue rectangle. The picture of B does not vanish.

Disclaimer: No resemblance to real celebrities and their pets is intended. This problem is gender neutral and species neutral. In my own mental image, B is a marsupial.