Review sheet for Exam 1 of Math 152

These problems are presented in order to help you understand the material that is listed prior to the first exam in the syllabus. DO NOT assume that your first midterm exam will resemble this set of problems. The following 20 problems are not meant to be a sample exam. These problems are just a study guide.

(1) Consider the solid whose base is the unit circle $x^2 + y^2 = 1$ and whose vertical cross sections perpendicular to the x-axis are rectanges of height $f(x) = |x|$. Find the volume of this solid.

(2) Find all values of c in the interval $[-1, 1]$ which have the following property: $f(c) =$ the average value of $f(x)$ on $[-1, 1]$ where $f(x) = \sqrt{1 - x^2}$. Which theorem guarantees the existence of at least one such c?

(3) Consider a right circular cone with height H and base of radius R. Find the volume of this cone in two different ways: (a) using disks, (b) using shells.

(4) Consider the triangular region in the xy-plane which is bounded by the lines $x = 3$, $y = 2$ and $y = 2(x - 3)$. A solid is obtained when this region is rotated about the line $x = 1$. Find the volume of this solid in two different ways: (a) using washers, (b) using shells.

(5) Evaluate the following integrals using substitutions:
(a) $\int \cot x \, dx$
(b) $\int \tan x \, dx$
(c) $\int \tan x \, dx$ using the substitution $u = \sec x$
(d) $\int \tanh x \, dx$
(e) $\int \coth x \, dx$

(6) (A) Evaluate $\int \csc x \, dx$ using the substitution $u = \csc x + \cot x$. (B) Evaluate $\int \csc x \, dx$ using the substitution $u = \csc x - \cot x$. (C) Show that these two answers are really equal.

(7) Evaluate $\int \sin^2 x \, dx$ and $\int \cos^2 x \, dx$ using integration by parts.

(8) Evaluate $\int \tan x \sec^4 x \, dx$ in two different ways.

(9) Evaluate $\int x \tan^{-1} x \, dx$.

More questions on the next page.

(10) Evaluate $\int (\ln x)^3 \, dx$.

(11) Let a and b be nonzero constants, where $a^2 \neq b^2$. Evaluate $\int \cos(ax)\cos(bx) \, dx$ using two integrations by parts.

(12) Evaluate $\int \sec^3 x \, dx$ using integration by parts.

(13) Evaluate $\int \sqrt{9 + x^2} \, dx$.

(14) Evaluate $\int \sqrt{9 - x^2} \, dx$.

(15) Evaluate $\int \dfrac{dx}{(1 + x^2)^2}$.

(16) Evaluate $\int \dfrac{3x^2 - 3x - 2}{(x^2 - 1)(x - 1)} \, dx$.

(17) Evaluate $\int \dfrac{x^2 + 3x}{(x^2 + 1)(x + 1)} \, dx$.

(18) Show that $\displaystyle\int_0^1 \frac{e^x}{x} \, dx$ diverges and $\displaystyle\int_1^\infty x e^{-x^4} \, dx$ converges.

(19) Evaluate $\displaystyle\int_4^\infty \frac{dx}{(2x + 1)(3x + 1)}$.

(20) Show that $\displaystyle\int_1^\infty \frac{\cos^2 x}{x^3} \, dx$ converges.