Solutions to Additional Review Problems for Math 152, Spring 2017

Problem (1): The length is

$$\begin{aligned}\int_0^{2\pi} \sqrt{r^2 + (dr/d\theta)^2}\, d\theta &= \int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + \sin^2\theta}\, d\theta = \int_0^{2\pi} \sqrt{2 - 2\cos\theta}\, d\theta \\ &= 2\int_0^{2\pi} \sqrt{\frac{1-\cos\theta}{2}}\, d\theta = 2\int_0^{2\pi} \sqrt{\sin^2(\theta/2)}\, d\theta \\ &= 2\int_0^{2\pi} |\sin(\theta/2)|\, d\theta = 2\int_0^{2\pi} \sin(\theta/2)\, d\theta = 8.\end{aligned}$$

Problem (2): The area is

$$\frac{1}{2}\int_0^{2\pi} (1+\sin\theta)^2\, d\theta = \frac{1}{2}\int_0^{2\pi} 1 + 2\sin\theta + \sin^2\theta\, d\theta = \frac{1}{2}(2\pi + 0 + \pi) = \frac{3\pi}{2}.$$

Problem (3):

For $r = \sin\theta$: Multiplying $r = \sin\theta$ by r, we get $r^2 = r\sin\theta$, which is $x^2 + y^2 = y$. This is $(x-0)^2 + (y-1/2)^2 = (1/2)^2$. The center of this circle is $(0, 1/2)$. The radius of this circle is $1/2$.

For $r = \cos\theta$: Multiplying $r = \cos\theta$ by r, we get $r^2 = r\cos\theta$, which is $x^2 + y^2 = x$. This is $(x-1/2)^2 + (y-0)^2 = (1/2)^2$. The center of this circle is $(1/2, 0)$. The radius of this circle is $1/2$.

Problem (4): The equation is equivalent to $r\cos(\theta + \pi/4) = 1/2$, which is

$$r\big(\cos\theta\cos(\pi/4) - \sin\theta\sin(\pi/4)\big) = 1/2.$$

This is the same as $(r\cos\theta)\sqrt{2}/2 - (r\sin\theta)\sqrt{2}/2 = 1/2$, which is $x(\sqrt{2}/2) - y(\sqrt{2}/2) = 1/2$. This is the equation of a line in the xy-plane.

Problem (5): The complex conjugate is $-2 + 23i$. The modulus is $\sqrt{533}$.

Problem (6):

$$\begin{aligned}e^{i\theta} &= 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \frac{(i\theta)^6}{6!} + \frac{(i\theta)^7}{7!} + \cdots \\ &= 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - i\frac{\theta^7}{7!} + \cdots \\ &= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right) \\ &= \cos\theta + i\sin\theta.\end{aligned}$$

Problem (7): $\sin(5x) = 5\cos^4 x \sin x - 10\cos^2 x \sin^3 x + \sin^5 x$.

Problem (8): $i\ ,\ -\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\ ,\ \dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\ .$