Formula Sheet for Math 151, Exam 1

Lines: If $(x_1, y_1), (x_2, y_2)$ lie on a line L, the slope of L is $m = \frac{y_2-y_1}{x_2-x_1}$ and the equation is $y - y_1 = m(x - x_1)$.
Distance: (x_1, y_1) to (x_2, y_2): $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Circle, center (a, b), rad. r: $(x - a)^2 + (y - b)^2 = r^2$.

In a right triangle: $\sin\theta = \frac{opp}{hyp}$, $\cos\theta = \frac{adj}{hyp}$, $\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1

Periodicity: $\sin(x + 2\pi) = \sin(x)$, $\cos(x + 2\pi) = \cos(x)$, $\tan(x + \pi) = \tan(x)$.
Identities: $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $\sin(2x) = 2\sin x\cos x$, $\cos(2x) = \cos^2 x - \sin^2 x$.
Addition: $\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$ $\quad\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$ $\quad\pi \approx 3.1416$.

Exponentials and logarithms: $a, b, t, u, y > 0$, r, v, w, x any real numbers: $a^{v+w} = a^v a^w$, $a^{vw} = (a^v)^w$, $a^{-v} = 1/a^v$, $a^0 = 1$, $(ab)^v = a^v b^v$, $\log_a(t) = \ln(t)/\ln(a)$. $e^x = y$ is equivalent to $x = \ln y$, $e^{\ln y} = y$, $\ln(e^x) = x$. $\ln(tu) = \ln(t) + \ln(u)$, $\ln(u^r) = r\ln(u)$, $\ln(1/u) = -\ln(u)$, $\ln(1) = 0$, $e \approx 2.718$.

Squeeze Theorem: If $f(x) \le g(x) \le h(x)$ near $x = a$ and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.

Intermediate Value Theorem: If f is continuous on $[a, b]$ and N is any number between $f(a)$ and $f(b)$, there is a number c in $[a, b]$, such that $f(c) = N$.
Corollary: If f changes sign from a to b, then $f(c) = 0$ with c between a and b.

Definition of the Derivative: $\quad f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$; $\quad f'(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$.

$f(x)$	$f'(x)$
c, const.	0
x^r	rx^{r-1}
e^x	e^x
$\ln x$	$1/x$

$f(x)$	$f'(x)$
a^x	$(\ln a)a^x$
$\log_a(x)$	$1/(\ln(a)\cdot x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x\cot x$

$f(x)$	$f'(x)$
$\sin^{-1}(x)$	$1/\sqrt{1-x^2}$
$\tan^{-1}(x)$	$1/(x^2+1)$
$\sec^{-1}(x)$	$1/(\lvert x\rvert\sqrt{x^2-1})$
$\cos^{-1}(x)$	$-1/\sqrt{1-x^2}$

Rules of Differentiation: $\frac{d}{dx}(cu) = c\frac{du}{dx}$, c a const., or $(cf)'(x) = cf'(x)$. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$, or $(f + g)'(x) = f'(x) + g'(x)$. Product Rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, or $(fg)'(x) = f(x)g'(x) + f'(x)g(x)$.
Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, or $(f/g)'(x) = (g(x)f'(x) - f(x)g'(x))/(g(x)^2)$.

Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, or $(f\circ g)'(x) = f'(g(x))g'(x)$. Replacing x by u and multiplying by $\frac{du}{dx}$, we can apply the Chain Rule to all boxed derivative formulas. Some examples are: $\frac{d}{dx}(u^r) = ru^{r-1}\frac{du}{dx}$, $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$, $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$, $\frac{d}{dx}(\sin u) = (\cos u)\frac{du}{dx}$, $\frac{d}{dx}(\cos u) = -(\sin u)\frac{du}{dx}$, $\frac{d}{dx}(\tan u) = (\sec^2 u)\frac{du}{dx}$.

Bodies in Free Fall: The distance above ground level of a body in free fall in the earth's atmosphere is $s(t) = s_0 + v_0 t - gt^2/2$, where s_0 is the position at time $t = 0$, v_0 is the velocity at time $t = 0$, and g is the acceleration due to gravity with $g = 32\text{ft/s}^2$ or $g = 9.8\text{m/s}^2$.