**Formula Sheet for Math 151, Exam 2**

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## Exponential and Logarithmic Function

$$e^{x+y} = e^x e^y, \qquad e^{x-y} = \frac{e^x}{e^y}, \quad e^{xy} = (e^x)^y, \quad \ln(xy) = \ln(x) + \ln(y), \quad \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^y) = y\ln(x), \quad \ln(e^x) = x, \quad e^{\ln(x)} = x, \quad a^x = e^{\ln(a)x}, \qquad \log_a(x) = \frac{\ln(x)}{\ln(a)}$$

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## Trigonometry

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}, \qquad \sec(x) = \frac{1}{\cos(x)}, \qquad \csc(x) = \frac{1}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1, \qquad \sin(2x) = 2\sin(x)\cos(x), \qquad \cos(2x) = \cos(x)^2 - \sin(x)^2$$

$$\sin(x)^2 = \frac{1}{2}\big(1 - \cos(2x)\big), \quad \cos(x)^2 = \frac{1}{2}\big(1 + \cos(2x)\big)$$

| $x$ | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\sin(x)$ | $0$ | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $1$ |
| $\cos(x)$ | $1$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | $0$ |
| $\tan(x)$ | $0$ | $\sqrt{3}/3$ | $1$ | $\sqrt{3}$ | $\pm\infty$ |

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## Geometry

**Distance:** $d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$

**Volume:** Cone/Pyramid: $\frac{1}{3}$ base area $\times$ height, Sphere: $\frac{4}{3}\pi r^3$

**Lines:** Point-point: $\dfrac{y - y_0}{x - x_0} = \dfrac{y_1 - y_0}{x_1 - x_0}$, Point-slope: $y - y_0 = m(x - x_0)$

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## Derivatives of some trigonometric functions

$$\frac{d}{dx}\tan(x) = \sec^2(x), \quad \frac{d}{dx}\sec(x) = \sec(x)\tan(x), \quad \frac{d}{dx}\cot(x) = -\csc^2(x), \quad \frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

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## Derivatives of some inverse trigonometric functions

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2 - 1}}, \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2 - 1}}$$