Formula Sheet for Math 151, Final Exam

Lines: If $(x_1, y_1), (x_2, y_2)$ lie on a line L, the slope of L is $m = \frac{y_2 - y_1}{x_2 - x_1}$ and the equation is $y - y_1 = m(x - x_1)$.
Distance, (x_1, y_1) to (x_2, y_2): $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
Circle, center (a, b), rad. r: $(x - a)^2 + (y - b)^2 = r^2$.

Trig: In a right triangle:
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1

Periodicity: $\sin(x + 2\pi) = \sin(x)$, $\cos(x + 2\pi) = \cos(x)$, $\tan(x + \pi) = \tan(x)$.
Identities: $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $\sin(2x) = 2\sin x\cos x$, $\cos(2x) = \cos^2 x - \sin^2 x$.
Addition: $\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$ $\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$ $\pi \approx 3.1416$.
Inverses: The **range** of $\sin^{-1} x$, $\tan^{-1} x$ and $\csc^{-1}$x is the subset of $[-\pi/2, \pi/2]$ avoiding values that correspond to $x = \infty$. The range of the other inverse trig functions is a similar subset of $[0, \pi]$.

Exponentials and logarithms: $a, b, t, u, y > 0$, r, v, w, x any real numbers: $a^{v+w} = a^v a^w$, $a^{vw} = (a^v)^w$, $a^{-v} = 1/a^v$, $a^0 = 1$, $(ab)^v = a^v b^v$, $\log_a(t) = \ln(t)/\ln(a)$. $e^x = y$ is equivalent to $x = \ln y$, $e^{\ln y} = y$, $\ln(e^x) = x$. $\ln(tu) = \ln(t) + \ln(u)$, $\ln(u^r) = r\ln(u)$, $\ln(1/u) = -\ln(u)$, $\ln(1) = 0$, $e \approx 2.718$.

Squeeze Theorem: If $f(x) \le g(x) \le h(x)$ near $x = a$ and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.

Intermediate Value Theorem: If f is continuous on $[a, b]$ and N is between $f(a)$ and $f(b)$, there is a number c in $[a, b]$, such that $f(c) = N$. **Corollary**: If f changes sign from a to b, then $f(c) = 0$ with c between a and b.

Definition of the Derivative: $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$; $f'(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$.

$f(x)$	$f'(x)$
c, const.	0
x^r	rx^{r-1}
e^x	e^x
$\ln x$	$1/x$

$f(x)$	$f'(x)$
a^x	$(\ln a)a^x$
$\log_a(x)$	$1/(\ln(a) \cdot x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x\cot x$

$f(x)$	$f'(x)$
$\sin^{-1}(x)$	$1/\sqrt{1-x^2}$
$\tan^{-1}(x)$	$1/(x^2+1)$
$\sec^{-1}(x)$	$1/(\lvert x\rvert\sqrt{x^2-1})$
$\cos^{-1}(x)$	$-1/\sqrt{1-x^2}$

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Rules of Differentiation: $\frac{d}{dx}(cu) = c\frac{du}{dx}$, c a const., or $(cf)'(x) = cf'(x)$. $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$, or $(f+g)'(x) = f'(x) + g'(x)$. **Product Rule**: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, or $(fg)'(x) = f(x)g'(x) + f'(x)g(x)$. **Quotient Rule**: $\frac{d}{dx}(u/v) = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right)/v^2$, or $(f/g)'(x) = (g(x)f'(x) - f(x)g'(x))/(g(x)^2)$.

Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, or $(f \circ g)'(x) = f'(g(x))g'(x)$. Replacing x by u and multiplying by $\frac{du}{dx}$, we can apply the Chain Rule to all boxed derivative formulas. Some examples are: $\frac{d}{dx}(u^r) = ru^{r-1}\frac{du}{dx}$, $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$, $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$, $\frac{d}{dx}(\sin u) = (\cos u)\frac{du}{dx}$, $\frac{d}{dx}(\cos u) = -(\sin u)\frac{du}{dx}$, $\frac{d}{dx}(\tan u) = (\sec^2 u)\frac{du}{dx}$.

Bodies in Free Fall: If air resistance is neglected, then the height of a body in free fall near the surface of the earth is $s(t) = s_0 + v_0 t - gt^2/2$, where s_0 is the position at time $t = 0$, v_0 is the velocity at time $t = 0$, and g is the acceleration due to gravity with $g = 32\text{ft/s}^2$ or $g = 9.8\text{m/s}^2$.

Linear or Tangent Line Approximation (or Linearization) of $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$.

Newton's Method to approximate a solution r of $f(x) = 0$. Choose a point x_0 close to r. Calculate the terms $x_0, x_1, x_2, x_3, \ldots$ of the sequence defined recursively by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

Rolle's Theorem: Suppose f is a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b) = 0$, then $f'(c) = 0$ for some c in (a, b).

Mean Value Theorem: Suppose f is a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Then there is a point c in (a, b) such that $f(b) - f(a) = f'(c)(b - a)$.

First Derivative Test: Suppose that f is a differentiable function and $f(c) = 0$. (a) If f' changes sign from $+$ to $-$ at $x = c$, a local maximum occurs at $x = c$. (b) If f' changes sign from $-$ to $+$ at $x = c$, a local minimum occurs. (c) If f' does not change sign at $x = c$, neither a local maximum or minimum occurs at $x = c$.

Second Derivative Test: Suppose that f is a twice differentiable function and $f'(c) = 0$. (a) If $f''(c) > 0$, a local minimum occurs at $x = c$. (b) If $f''(c) < 0$, a local maximum occurs. (c) If $f''(c) = 0$, the test fails.

L'Hôpital's Rule: If $\displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\displaystyle\frac{\pm\infty}{\pm\infty}$, then $\displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$. (Here, a may be a finite pt. or $\pm\infty$.)

Integration or anti-differentiation: $\int f(x)\,dx = F(x) + C$ means that $F'(x) = f(x)$. Formulas can be found by reversing the differentiation formulas: $\int x^r\,dx = x^{r+1}/(r+1) + C$, if $r \neq -1$ and $\int x^{-1}\,dx = \ln|x| + C$.