Review Problems for the first exam in Math 151 Spring 2009.
NOTE : These are only practice problems!

1. Find the equation of the line that passes through $(2, 4)$ and is perpendicular to the line $2x + 3y = 12$.

2. Let $f(x) = \sqrt{x^2 + 2x - 15}\,, \quad g(x) = \dfrac{1}{x}$
a) Find the domain of the function $f(x)$.
b) Find $g(f(x))$ and $f(g(x))$.
c) Find the domains of $g(f(x))$.

3. Find the **exact** value of each of the following limits. Show all work and/or give reasons for your answers:
a) $\displaystyle\lim_{x\to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$ b) $\displaystyle\lim_{x\to 4} \frac{x^2 - 5x + 6}{x^2 - 4}$ c) $\displaystyle\lim_{x\to 0} \frac{\tan 2x}{\tan 7x}$
d) $\displaystyle\lim_{x\to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x}$ e) $\displaystyle\lim_{x\to 3} \frac{|x - 3|}{x - 3}$ f) $\displaystyle\lim_{x\to 0} \frac{1 - \cos x}{x^2}$

4. Let $g(x) = x^5 + x^3 + 30$. **Without graphing** the function g, use a theorem to show that there is at least one number $c \in (-2,\ 2)$ such that $g(c) = 0$.
HINT: don't try to find c !

5. Let

$$f(x) = \begin{cases} x^2 + 1 & x > 2 \\ A & x = 2 \\ 2x + 1 & 2 > x \geq 0 \\ x^2 + 3 & x < 0 \end{cases}$$

a)For what value of A is f continuous at $x = 2$? **Explain!**
b)Find the following limits or write DNE if the limit doesn't exist. **Show all work.**
$\lim_{x\to 2} f(x), \quad \lim_{x\to 1} f(x), \quad \lim_{x\to 0} f(x), \quad \lim_{x\to(-1)} f(x)$
b) Is $f(x)$ differentiable at $x = 0$?

6. Find the following derivatives from the definition:
a) $f(x) = x^2 + 3x$ b) $g(x) = \dfrac{1}{x + 2}$ c) $h(x) = \sqrt{x - 3}$

7. The line $y = 2x + 3$ is tangent to the parabola $y = x^2 + B$. Find B.

8. Find the derivative of the following functions. Don't simplify!
a) $f(x) = \dfrac{7}{x^{3/7}} + \sqrt{x^5} + x^7 + 45$ b) $g(x) = (x + 9) * (x^2 - 7x)$
c) $h(x) = \left(\dfrac{x^2 + 7}{x^5 - 8x}\right)^9$ d) $k(x) = \dfrac{(x^4 + 2)^6}{\sqrt[3]{x^3 + 5x}}$ e) $\ln x^5 + x - \ln x$

9. Find the equation of the tangent line for the graph of $f(x) = 2 * \sqrt{x} + x^2 - 5$ at $x = 1$.

10. Let $f(x) = g(\sqrt{x + 3})$. Find $f(6)$ and $f'(6)$.
It is impossible to find g, but it will be useful to use some of the following known values of $g(x)$ and $g'(x)$:
$g(1) = 2$, $g(2) = 5$, $g(3) = 7$, $g(4) = 2$, $g(5) = 11$, $g(6) = 13$ and $g(7) = 21$
$g'(1) = 3$, $g'(2) = 2$, $g'(3) = 8$, $g'(4) = 10$, $g'(5) = 12$, $g'(6) = 21$ and $g(7) = 23$.

11. Sketch a possible graph of F on $[-3,\ 3]$ such that:
F is continuous on $[-3,\ 0)$ and $(0,\ 3]$, $\lim_{x\to 0^+} F(x) = 5$, $\lim_{x\to 0^-} F(x) = -2$, F is not differentiable only at $x = 0$ and $x = 1$.

12. Let $y = 2x^4 + 3x^2 + 12$. Find $\dfrac{d^3y}{dx^3}$.

13. The distance s (in feet) covered by a car t seconds after starting from rest is given by $s(t) = 20t + 6t^2 + t^3$, when $0 \le t \le 20$.
a) What is the velocity of the car 5 seconds after starting from rest?
b) What is the acceleration of the car at that time?

14. Sketch a possible function on the domain $(-2\ ,\ 4)$ that is :
Not differentiable only at $x = (-1.5)\ ,\ (-1)\ ,\ 0\ ,\ 0.5\ ,\ 1\ ,\ 3,$ not continuous only at $x = (-1)\ ,\ 0.5\ ,\ 3$ and has no limit only at $x = 0.5\ ,\ 3$.

15. Let $h(x) = f(g(x))$. Assume that $f(1) = 2,\quad f'(1) = 7\quad f(2) = 5\quad f'(2) = 5,\quad g(1) = 2\ \ and\ \ g'(1) = 3$. Find $h'(1),\ (fg)'(1),\ (f/g)(1),\ f(g(1))$.

16. Suppose that f and g are differentiable functions such that $f(g(x)) = 8x^2$ to all real numbers x. Assume that $f(2) = 7,\quad g(2) = 4,\quad f'(2) = 4\quad and\quad f'(4) = 2$. What is $g'(2)$?

17. Expend the following expression: $\ln \dfrac{\sqrt[5]{x}x^3y^2}{6\sqrt{y}x^9}$