**Math 151, Spring 2009, Review Problems for Exam 2**

**Your first exam is likely to have problems that do not resemble these review problems.**

1. A function $y = f(x)$ is defined implicitly by the equation $x^2 + 3xy + y^2 = 5$.
   (a) Find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.
   (b) Find the equation of the tangent line to the curve at the point $(1, 1)$.
   (c) Show that no point on the graph of $x^2 + 3xy + y^2 = 5$ has a horizontal tangent line.

2. Find the points on the graph of $y^2 = x^3 - 9x + 1$ (Figure 1) where the tangent line is horizontal.

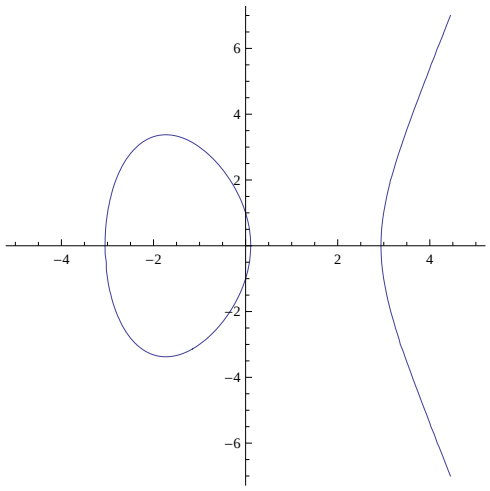


Figure 1: Graph of $y^2 = x^3 - 9x + 1$.

3. Differentiate the following functions.
   (a) $\dfrac{\tan^{-1}(x^2)}{3x^2 + 3}$ (b) $x^{\cos(x)}$ (c) $2^x \ln\left(\sin^{-1}(x)\right)$ (d) $\log_2\left(\cos^{-1}(x^2 + 1)\right)$

4. A rocket travels vertically at a speed of 600 mph. The rocket is tracked through a telescope by an observer located 10 miles from the launching pad. Find the rate at which the angle between the telescope and the ground is increasing 1 min after lift-off.

5. For each of the functions below, find the local maxima, local minima, inflection points, asymptotes and the intervals where is it increasing, decreasing, concave up and concave down. Draw the graph.
   (a) $\dfrac{x}{5x - 2}$ (b) $\dfrac{\ln(x)}{x}$ (c) $4x^3 - 6x^2$ (d) $\dfrac{x^2 - 1}{x^2 - 4}$

6. Let $f(x) = \sqrt{1-3x}$. Use the Linear Approximation of $f(x)$ at $c = -8$ to compute an approximation for $f(-9)$.

7. Estimate $\tan^{-1}(1.01) - \tan^{-1}(1)$ using Linear Approximation.

8. Let $f(x) = 2\sin(x) + \cos(2x)$.

   (a) Find the critical points of $f(x)$ on $[0, \pi]$.

   (b) Find the maximum and minimum values of $f(x)$ on the interval $[0, \pi]$.

9. Use Newton's Method to approximate $2^{-1/2}$. To do so, follow the next steps:

   (a) Find a function $f(x)$ for which $2^{-1/2}$ is a root.

   (b) Set $x_0 = 1$.

   (c) Find $x_2$.

10. Calculate the following limits.

    (a) $\lim_{x\to 1} \ln(x)\tan\left(\frac{\pi x}{2}\right)$ (b) $\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^{2x}$ (c) $\lim_{x\to 0} \frac{\sin(x) - x\cos(x)}{x - \sin(x)}$

    (d) $\lim_{x\to 0} \frac{x - \sin(x)}{x^3}$ (e) $\lim_{x\to 0} (1+x)^{1/x}$ (f) $\lim_{x\to 0} \frac{3x - \tan^{-1}(3x)}{x^3}$

    (g) $\lim_{x\to\infty} \left(1 + \frac{1}{3x}\right)^x$

11. Suppose that $f(x)$ is a differentiable function, $f(0) = 6$, and $f'(x) \geq 3$ for all $x$. Show that $f(\pi^2) \geq 3\pi^2 + 6$.

12. Find the point on the parabola $y = x^2$ which is closest to the point $(0, 2)$.

13. Find the dimensions that maximize the volume of a cylinder (top and bottom included) with surface area 400 ft$^2$.

14. Compute the area under the graph of $f(x) = 3x$ over $[0, 2]$ using $\lim_{N\to\infty} R_N$.

15. Solve the differential equation

$$y'' = \cos(x)$$

    with initial conditions $y(\pi) = \pi$ and $y'(\pi) = 1$.