Math 151, Spring 2009, Review Problems for Final Exam

This review sheet emphasizes the material covered after Exam 2. You SHOULD also study the problems in the review sheets for the first and second exam.

1. Evaluate the following integrals:
   (a) $\int x^2\sqrt{4x+3}\,dx$ (b) $\int \frac{e^{-x}}{1+e^{-2x}}\,dx$ (c) $\int (10x+3)^{20}\,dx$
   (d) $\int \frac{1}{x(\ln(x))^2}\,dx$ (e) $\int \sqrt{x}(x^4+1)\,dx$

2. Evaluate the following integrals:
   (a) $\int_{-3}^{2} |x^2+x-2|\,dx$ (b) $\int_0^{\ln(2)} \frac{e^x}{e^x+2}\,dx$ (c) $\int_0^{\frac{\pi}{4}} \left(\tan^3(x)+\tan(x)\right)\,dx$
   (d) $\int_1^2 \frac{\sqrt{x}+6}{x}\,dx$

3. Assume $f(x)$ is a continuous function on $[4,7]$ with the property $\int_1^2 f(3x+1)\,dx = 9$. Find $\int_4^7 f(x)\,dx$.

4. Let
$$f(x) = \begin{cases} 4x+3, & x<2 \\ x^3+x+1, & x\geq 2. \end{cases}$$
   Find $\int_0^3 f(x)\,dx$.

5. Evaluate the following derivatives:
   (a) $\frac{d}{dx}\left(\frac{e^{2x}\ln(4x)}{x+\sin^{-1}(x)}\right)$ (b) $\frac{d}{dx}\tan^{-1}(x+3^x)$ (c) $\frac{d}{dx}\int_0^x e^{-t^2}\,dt$
   (d) $\frac{d}{dx}\int_{-x}^{x} \frac{1}{\sqrt{t^4+1}}\,dt$ (e) $\frac{d}{dx}\sqrt{\sec(x^2)+x^2+1}$

6. Evaluate $\sum_{j=0}^{N}(3j+1)^2 = 1^2+4^2+7^2+\cdots+(3N+1)^2$.

7. Find the left endpoint, midpoint, and right endpoint approximations, L_4, M_4, R_4, for $f(x) = \frac{1}{x+3}$ on $[0,1]$.

8. Find the horizontal and vertical asymptotes of the following functions:
   (a) $\frac{x^2}{4x^2-2}$ (b) $\frac{\ln\left((x^2-4)^2\right)}{x^2-1}$ (c) $\frac{e^x+3e^{-x}}{4e^{2x}-9e^{-x}}$

9. The radius of a right circular cone of fixed height $h = 20$ cm is increasing at a rate of 2 cm/s. How fast is the volume increasing when $r = 10$.

10. Consider the function $f(x) = x^2 \ln(x)$. Find the local maxima, the local minima, infection points, the intervals where it is increasing, decreasing, concave up, and concave down.

11. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = \frac{1}{\sqrt{x}}$.

12. Find $\frac{dy}{dx}$ for $x^2y^3 + 3xy + 3y = x^2 + 1$.

13. Find the tangent line for the curve at the given point:

$$xe^y + yx^2 = 1, \quad (x, y) = (1, 0).$$

14. Find the following limits:
(a) $\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x - 3}$ (b) $\lim_{x\to 0} \frac{\tan^{-1}(4x)}{\tan(3x)}$ (c) $\lim_{x\to 2} \frac{x^2 - x - 2}{x^3 + x^2 - 6x}$
(d) $\lim_{x\to\infty} \left(\frac{2x}{2x+1}\right)^x$ (e) $\lim_{x\to 0^+} x\ln(x)$.

15. Find the second derivative of the following functions:
(a) $\sin^{-1}(x^2)$ (b) x^2e^{-3x} (c) $2^{\sin(x)}$.

16. Find the global maximum and global minimum of $f(x) = 2 - 9x - 3x^2 + x^3$ on $[-2, 5]$.

17. Assume $f(x) = \sin(g(x))$. If $g'(0) = 3$ and $g(0) = \pi/4$, find $f'(0)$.

18. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30/ft and on the other three sides by a metal fence costing \$10/ft. If the area of the garden is 1000 ft^2, find the dimensions of the garden that minimize the cost.

19. Find the values of a and b that make the function

$$f(x) = \begin{cases} 3x^2 + 1, & x < 1 \\ ax + b, & 1 \le x \le 2 \\ x^3, & x > 2 \end{cases}$$

continuous everywhere.