(10) 1. Find an equation for the function $f(x)$ if $f''(x) = 6x + 4$ and $f(2) = 1$, $f'(2) = -1$.

(15) 2. A differentiable function $y = f(x)$ has the following properties:

(1) The domain is $(-\infty, \infty)$.

(2) The only x-intercept is $x = 0$.

(3) $\lim_{x \to \infty} f(x) = -\infty$ and $\lim_{x \to -\infty} f(x) = 0$.

(4) $f(x)$ is negative for $x > 0$ and $f(x)$ is positive for $x < 0$. Moreover, $f(1) = -9$, $f(-2) = 2$, and $f(-1) = 3$.

(5) $f'(x)$ is 0 only at $x = -1$. Moreover, $f'(-4)$ is positive while $f'(4)$ is negative.

(6) $f''(x)$ is 0 only at $x = -2$. Moreover, $f''(-6)$ is positive while $f''(4)$ is negative.

(a) Determine the intervals where the function is increasing and the intervals where it is decreasing.

(b) Determine the x-values where the function has a local maximum and those where it has a local minimum.

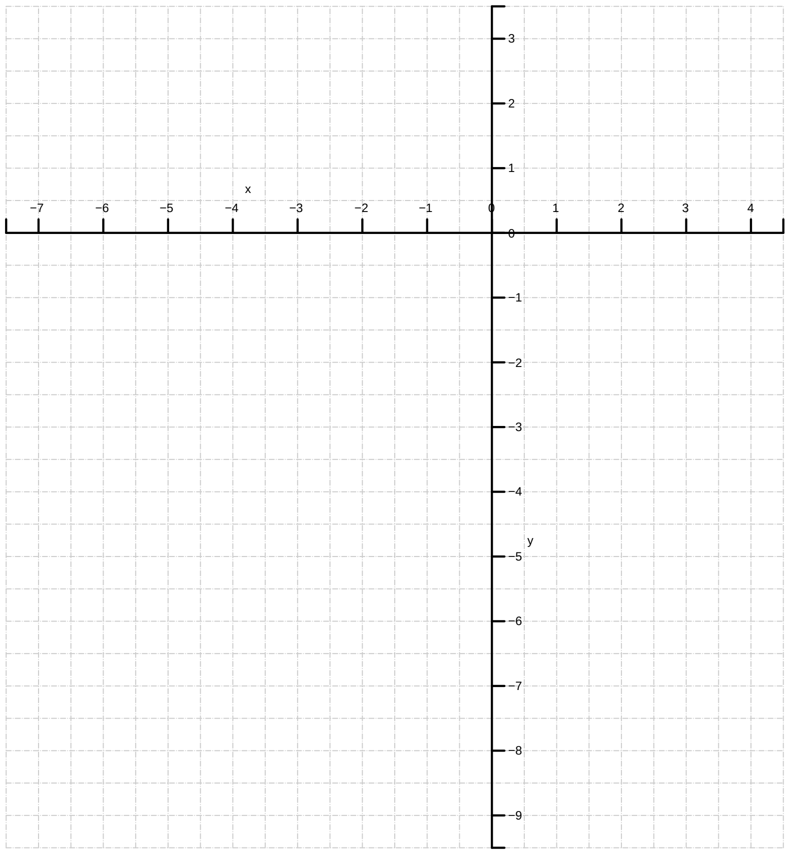
(c) Determine the intervals where the function is concave up and the intervals where it is concave down.

(d) Determine the x-values where the inflection points occur.

(e) Determine all vertical and horizontal asymptotes.

(f) Sketch the graph of a function which has the properties found in parts (a)-(e) above. Show on the graph all intercepts, asymptotes, local maxima and local minima, and points of inflection.

B2



(20) 3. Compute $\dfrac{dy}{dx}$ in each of the following cases. You don't have to simplify.

(a) $y = \sqrt{3 + \dfrac{5}{x^9}}$

(b) $y = x^5 \tan^{-1}(8 - 7x)$

(c) $y = \dfrac{\cos(5 \ln x)}{7 + e^{8x}}$

(d) $y = \displaystyle\int_6^x \frac{t^3}{8 + t^4}\, dt$

(12) 4. Consider the function $f(x) = e^{3x}$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$ at the point $P = (1, e^3)$.

(b) Use linear approximation to get an estimate for the value of $f(1.1)$.

(c) Use the second derivative to determine whether your estimate for (b) is likely to be high or low.

(12) 5. Consider the equation $x^3 - 7x + 1 = 0$.

(a) Does this equation have a solution in the interval $[-3, -1]$? Justify your answer.

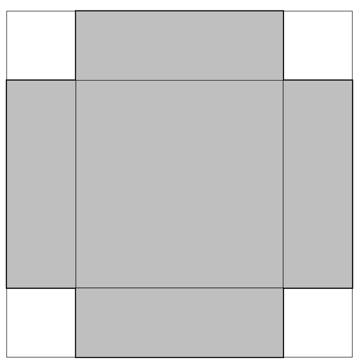
(b) Compute, using Newton's method, the second approximation of a solution by starting with the first approximation $x = -3$. You do not need to simplify your final numerical answer.

(10) 6. Using the limit definition of the derivative, find the derivative of $f(x)$ at $x = 2$ where

$$f(x) = x^2 - 2x + 1.$$

(10) 7. Find an equation of the line tangent to the graph of the function given implicitly by the equation $2x^2y - 3xy^2 = 14$ at the point $(-2, 1)$.

(10) 8. An open-top box is to be made by cutting small congruent squares from the corners of a 6 cm × 6 cm sheet of metal and bending up the sides. What is largest possible volume of such a box?



(20) 9. Compute the following limits.

(a) $\lim_{x \to 2^+} \frac{x^2 - 4}{|x - 2|}$

(b) $\lim_{x \to 1} \frac{x^2 - 4x + 5}{x^2 - 2}$

(c) $\lim_{x \to \infty} \frac{\cos(x)}{1 + x^2}$

(d) $\lim_{x \to 0} \frac{\cos(x) - 1}{e^x + e^{-x} - 2}$

(12) 10. Find the global maximum and minimum values of the function

$$f(x) = x^3 - 12x$$

on the interval $[-1, 3]$. You don't have to sketch the graph but you must justify your answer.

Global maximum of f: ______________ Global minimum of f: ______________

(15) 11. Evaluate the following integrals.

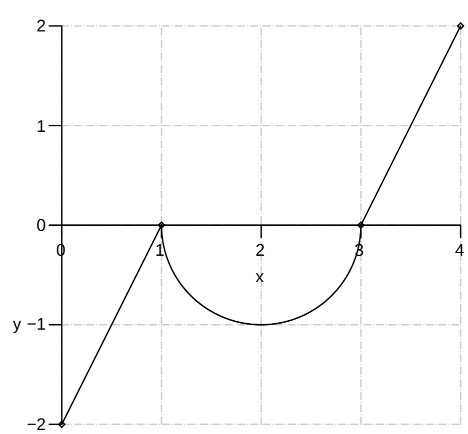
(a) $\displaystyle\int_0^1 \frac{e^x}{\sqrt{1+e^x}}\,dx$

(b) $\displaystyle\int \left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)^2\,dx$

(c) $\displaystyle\int_1^2 \frac{2x^2-1}{x}\,dx$

(10) 12. A car travels on a restricted-access highway which is 100 miles long. The speed limit on the road is 60 miles per hour. Suppose that $s(t)$, the car's distance from the beginning of the road, is a differentiable function of the time t. If the car enters the road at 9am and leaves the road at 10:15am at the other end of the highway, one and a quarter hours later, explain why the driver should expect to receive a speeding citation (ticket). Your explanation should be in complete English sentences. You should mention a specific result from this course and explain precisely why this result shows that the ticket is valid.

(12) 13. Let $f(x)$ be a function defined over the interval [0,4], whose graph is shown as below. The graph of f over [1,3] is a semi-circle centered at $(2, 0)$ and of radius 1, the other pieces are straight lines.



(a) Find $\int_0^4 f(x)\,dx$ geometrically.

(b) Compute the Riemann sum for $f(x)$ where $[0,4]$ is subdivided into 4 subintervals of equal size and the sample points are the right endpoints.

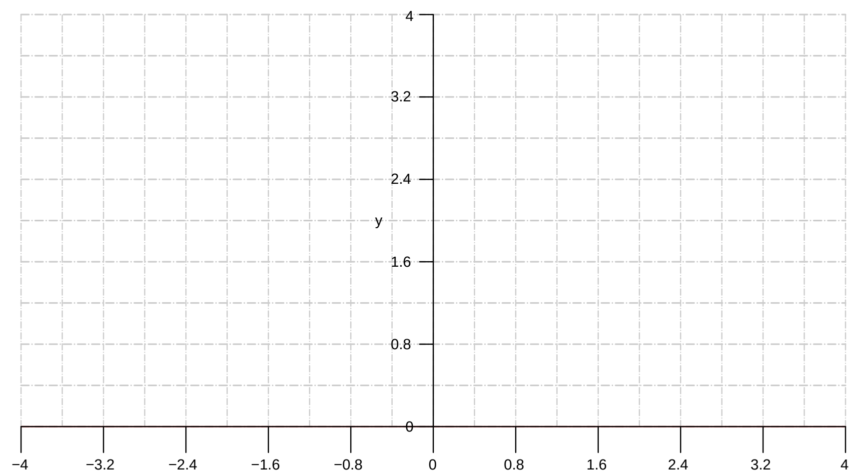
(c) Let $F(x) = \int_0^x f(t)\,dt$. Find $F'(2)$ and $F'(3)$.

(10) 14. Determine the values of the parameters a and b such that the following function $f(x)$ becomes continuous and differentiable at $x = -1$:

$$f(x) = \begin{cases} x^2 + 3x + b & \text{for } x > -1 \\ ax & \text{for } x \le -1 \end{cases}$$

(12) 15. Consider the finite region A bounded by the "wedge" $y = |x|$ and the parabola $y = \frac{1}{4}x^2 + 1$.

(a) Draw a sketch of the region A. [*Hint:* The parabola fits "snuggly" into the wedge.]



(b) Express the area of A in terms of one or more definite integrals.

(c) Compute the area of the region A as a sum of fractions (you do not need to simplify the answer).

(10) 16. The minute hand of a large tower clock is 2 m long. At each full hour, the tip T of the hand points to the center C of the numeral XII. How fast is the distance between T and C changing when it is 9 : 20 pm? [*Hint:* Sketch the triangle formed by C, T, and the center of the clock. Observe that it is isosceles.]

FINAL EXAM for MATH 151 **B**

December 18, 2006

Section number: ⌊_⌊_⌋ **Instructor:** ⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌊_⌋

Last Name, First Name: ⌊_⌋

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(SIGNATURE)

Show all your work. Full credit may not be given for an answer alone.

No books, no notes, no electronic devices.

Problem Number	Possible Points	Points Earned:
1	10	
2	15	
3	20	
4	12	
5	12	
6	10	
7	10	
8	10	
9	20	
10	12	
11	15	
12	10	
13	12	
14	10	
15	12	
16	10	
Total Points Earned:		