Math 151 Sample exam #1

Do all problems, in any order.

1. (15 points) f and g are differentiable functions about which the following is known:

$$f(3) = 2 \qquad f'(3) = 7 \qquad g(3) = 2 \qquad g'(3) = 5$$

Carefully compute the following quantities (an answer alone will *not* receive full credit):

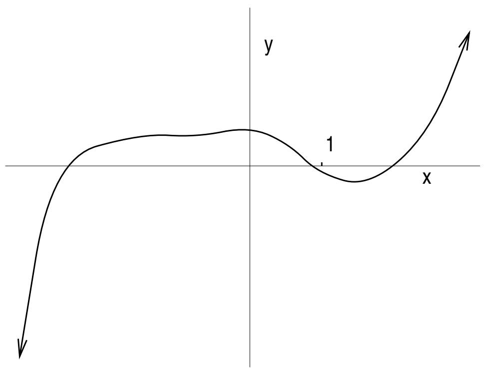
$$(f+g)'(3) \qquad\qquad (f \cdot g)'(3) \qquad\qquad \left(\frac{f}{g}\right)'(3)$$

2. (15 points) a) Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \dfrac{1}{x^2}$.

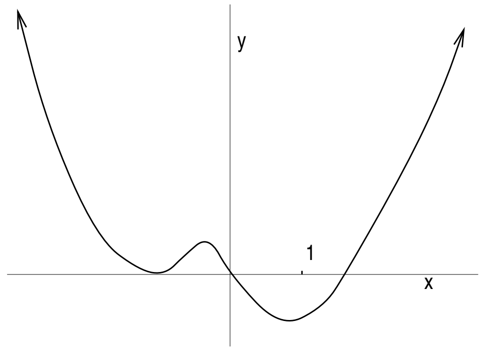
b) Use your answer to a) to find an equation for the line tangent to the curve $y = \dfrac{1}{x^2}$ when $x = 2$.

c) Sketch the graph of $y = \dfrac{1}{x^2}$ *and* the line you found in answer to b).

3. (15 points) Here are graphs of the functions F and G:



Graph of $y = F(x)$



Graph of $y = G(x)$

One of these functions is the derivative of the other function. Identify which is the derivative and explain your answer as well as you can with reference to the graphs (which you may wish to label). Your answer should be written in complete English sentences with as much evidence as possible given to support your assertion.

4. (20 points) Find the approximate location of a point on the curve $y = .5x + \sin x$ so that the line tangent to the curve at that point also passes through $(3, 3)$. Explain how you got your result.

5. (20 points) Consider the function

$$G(x) = Ax^3 + x^2 + \sin(Ax) + 1$$

Note that the parameter A appears in *two* places in the formula for G.

a) If $A = 0$ verify that G is *never* 0.

b) If $A > 3$ explain why $G(x) = 0$ *must* have a root in $[-1, 1]$. (Hint: consider G's values at the endpoints of the interval.)

c) Locate one approximate solution with accuracy $\pm .01$ of the equation $G(x) = 0$ in the interval $[-1, 1]$ when $A = 5$.

6. (15 points) Let $F(x) = \dfrac{x^2 - 1}{|x - 1|}$.

a) Find

i) $\lim_{x \to 1^+} F(x)$ ii) $\lim_{x \to 1^-} F(x)$

b) Does $\lim_{x \to 1} F(x)$ exist?

c) Sketch the graph of F.