Problems, Week 1

Please write solutions to two of these problems and hand them in next Monday. These written solutions should be of high-quality, with complete explanations given. While I encourage you to discuss the problems with other students and with your instructors, the written work you hand in must be your own explanations of the problems you choose.

1. a) What is the domain of the function H defined by the formula $H(x) = \sqrt{\dfrac{x}{4-x}}$?

b) A function is a **monomial** if it is a constant multiplied by an integer power of the independent variable (so $56x^4$ is a monomial). A function is a **polynomial** if it is a sum of monomials (so $2x^7 - 5.6x^{33} + 4\pi$ is a polynomial). A function is a **rational function** if it is a quotient of polynomials (so $\dfrac{4x^7 - x}{x^4 - 3x + .034}$ is a rational function). A function is an **algebraic function** if, in addition, roots are allowed (so $6x^{1/3} - \sqrt{\dfrac{2+x}{4-x^2}}$ is an algebraic function). Can you find an algebraic function whose *domain* is exactly those numbers x satisfying either $2 \le x < 3$ or $5 \le x < 7$? In mathematical notation, the domain of the function should be $[2,3) \cup [5,7)$. Here *domain* means the natural domain of the function, which is the collection of all numbers for which the formula defining the function makes sense.

c) Suppose that root extractions cost \$1, and additions and multiplications cost 5 cents, and divisions cost 10 cents. Thus the function H above would cost \$1.15. How much would your answer to part b) above cost? Can you find a cheaper answer?

2. Find the ranges of

$$H(x) = \sqrt{\frac{x}{4-x}} \text{ and } K(x) = \sqrt{x} - \sqrt{4-x} \text{ and } J(x) = \sqrt{x} + \frac{1}{\sqrt{4-x}}.$$

Verify your assertions.

3. If an example of any of the following exists, describe such an example and explain why it fulfills the requirements and sketch a graph of the example. If an example does not exist, explain why no such example exists.

a) A polynomial whose roots are exactly the numbers 1, -2 and 3.475.

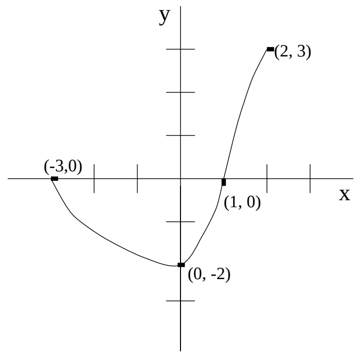
b) A polynomial whose roots are exactly the numbers 1, -2 and 3.475 and whose non-zero values are always positive.

c) A polynomial whose roots are exactly the numbers 1, -2 and 3.475 and which has degree $100,000$.

d) A polynomial whose roots are exactly the numbers 1, -2 and 3.475 and which has degree $100,000$ and whose non-zero values are always positive.

e) A polynomial whose roots are exactly the numbers 1, -2 and 3.475 and which has degree 57 and whose non-zero values are always positive.

4. Here is a graph of $y = A(x)$:



a) What are the domain and range of A?

b) Suppose B is the function defined by $B(x) = A(x) + 1$. Sketch the graph of B as well as you can. What are the domain and range of B?

c) Suppose C is the function defined by $C(x) = A(x)^2$. Sketch the graph of C as well as you can. What are the domain and range of C?

d) Suppose D is the function defined by $D(x) = \dfrac{1}{A(x)}$. Sketch the graph of D as well as you can. What are the domain and range of D?

5. a) Fill in the boxes below with the numbers 10^{-10}, -10^{10}, 10^{10}, and -10^{-10} so that the resulting statement is true.

$$\boxed{} < \boxed{} < \boxed{} < \boxed{}$$

b) For which numbers, w, is $w < \dfrac{1}{w}$? For which numbers, w, is $w < w^2$? For which numbers, w, is $w < -w$? For which numbers, w, is $-w < \dfrac{1}{w}$?

c) For which numbers, w, is $w^2 - 5w + 2 < 3w^4 - 17w^3 + 6w^2 + w + 1$?

6. a) Sketch a graph of $y = \big|x - |x - 3|\big|$.

b) Give a "piecemeal" definition of the function whose graph was sketched in a) *without* mentioning absolute value. The graph may help to answer this question, but you should justify your answer algebraically with a case-by-case argument from the equation for y (of the sort "when $x \geq 3$ then y is given by the formula ...").