Problems, Week 12

1. a) Find the dimensions of the rectangle of largest area that can be inscribed in the region bounded by the x-axis and the graph of $y = 1 - x^2$.

b) Suppose a and b are positive constants. Find the dimensions of the rectangle of largest area that can be inscribed in the triangle bounded by the x-axis, the y-axis, and the line $\frac{x}{a} + \frac{y}{b} = 1$.

2. What are the dimensions of the lightest cylindrical aluminum can with volume 1000 cm^3?

3. A farmer has 100 meters of fencing and wishes to use it to enclose three equal and contiguous pens, using an existing building as a common side of the three pens. What are the dimensions of the pen arrangement which gives the greatest total area inside the pens?

4. An island is at point A, 6 miles off shore from the nearest point B on a straight beach. A store is located at point C, which is 7 miles from B along the shore. If a person can row at the rate of 4 mph and can run at the rate of 5 mph, to what point on the shore should she row in order to get to the store as fast as possible?

5. a) Find three distinct functions $f(x)$ such that $f'(x) = 0$ for all x.

b) Find three distinct functions $g(x)$ such that $g'(x) = 4x$ for all x.

c) Find three distinct functions $h(x)$ such that $h'(x) = 3x - x^2$ for all x.

6. A charged particle moves along the x-axis under the influence of an electric field. The field strength varies with time, and as a result the velocity of the particle seems complicated. Let the position of the particle at time t be denoted by $x = x(t)$, and the velocity of the particle at time t be denoted $v = v(t)$.
Suppose $x(0) = 0$, and

$$v(t) = \begin{cases} 2t - 1, & \text{if } 0 \le t \le 1 \\ 4t - 3, & \text{if } 1 \le t \le 2 \\ 6t - 7, & \text{if } 2 \le t \le 3 \end{cases}$$

What is $x(1)$? What is $x(2)$? What is $x(3)$?
Sketch the graphs of $x = x(t)$ and $v = v(t)$.

7. Find a function $f(x)$ such that $f''(x) = 6x$ and the graph of $y = f(x)$ passes through the point $(0, 1)$ and has a horizontal tangent there. DO these conditions determine f completely?