# Problems, Week 6

1. Suppose that $f$ and $g$ are differentiable functions and that $h(z) = f(g(z))$. Fill in the gaps in the following table:

| $a$ | $g(a)$ | $g'(a)$ | $f(a)$ | $f'(a)$ | $h(a)$ | $h'(a)$ |
|---|---|---|---|---|---|---|
| 0 | 1 | $-3$ | 3 | $-2$ | $-1/3$ | ... |
| 1 | 2 | 0 | ... | $-7$ | ... | ... |
| 2 | 2 | $-1$ | $-1$ | ... | ... | 3 |

2. Suppose that $f$ is a differentiable function such that $f(x^5 + x + 1) = x^2$ for all $x$. Do *not* try to find a formula for $f(x)$ (there is no simple one), but show that it is possible to calculate $f'(3)$ nevertheless.

3. If $f(x) = u\big(v(x)\big)$, obtain a formula for $f''(x)$ in terms of $u(x)$, $v(x)$, and their first and second derivatives.

4. A particle is moving along the parabola $y = x^2$.

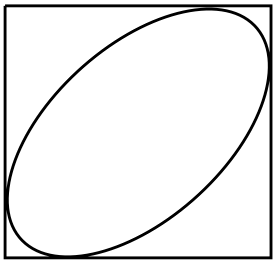
a) At the time the particle passes through the point $(3, 9)$, its $y$-coordinate is changing so that $\dfrac{dy}{dt} = 7$. How fast is its $x$-coordinate changing at that instant?

b) If the particle moves so that $\dfrac{dy}{dt} = 7$ for all $t$, what happens to $\dfrac{dx}{dt}$ as $t \to \infty$? If on the other hand $\dfrac{dx}{dt} = 7$ for all $t$, what happens to $\dfrac{dy}{dt}$ as $t \to \infty$?

5. The textbook states in a homework problem (# 41 of section 2.6) that

$$x^2 - xy + y^2 = 3$$

is the equation of a "rotated ellipse." I think the ellipse looks like this:



What are the dimensions and the location of the box containing the ellipse?

Note: the sides are vertical and horizontal and also tangent to the ellipse. Maybe you could find the slopes of lines tangent to the ellipse and check which lines are either horizontal or vertical.

6. If a spherical raindrop is evaporating at a rate proportional to its surface area, what can be said about the rate at which the radius is decreasing?

7. $BCDE$ is a rectangle, with $BC$ of length 1 inch, and $BE$ of length 2 inches. The point $Q$ moves from $D$ to $E$ at the uniform rate of 1 inch/minute, starting from $D$ at time $t = 0$. The point $P$ moves from $C$ to $D$ at the same rate, starting from $C$ at time $t = 0$.

a) During the one minute period beginning at time $t = 0$, what is the smallest that the area of the triangle $BPQ$ is, and what is the largest? (Hint: there are three right triangles in the picture.)

b) When $t = \frac{1}{2}$, is the length of $PQ$ increasing or decreasing? How fast?