Problems, Week 8

1. Suppose the richest x percent of people in the world have $10\sqrt{x}$ percent of the wealth. Then y percent of the wealth is held by ______ percent of the people. Fill in the blank; explain. (This question is from Gilbert Strang's text, *Calculus*.)

2. a) Suppose f is one-to-one and g is one-to-one, and the range of g is contained in the domain of f (so the composition $f \circ g$ is defined). If $F = f \circ g$, explain why F is one-to-one.

c) Suppose f and g both are one-to-one, are defined on intervals, and that $F = f \circ g$ is defined. Suppose you also know the following numerical information:

$$g(3) = 7 \qquad f(7) = 10 \qquad g'(3) = 2 \qquad f'(7) = 5$$

Compute: $F(3)$, $g^{-1}(7)$, $f^{-1}(10)$, $F^{-1}(10)$, $\left(g^{-1}\right)'(7)$, $\left(f^{-1}\right)'(10)$, and $\left(F^{-1}\right)'(10)$.

3. It is true that $Q(x) = x^5 + x^3 + x$ is a one-to-one function whose domain and range are all numbers. Suppose that R is the function inverse to Q. Compute $R(3)$, $R'(3)$ and $R''(3)$.

4. Suppose $Z(x) = \dfrac{3x-5}{2x-3}$.

a) Compute Z' and evaluate $Z(1)$ and $Z'(1)$.

b) Verify that Z is one-to-one.

c) Find W, the function inverse to Z, and evaluate $W(2)$ and $W'(2)$

d) Explain.

5. Graph $F_1(x) = \dfrac{\ln(1+x^2)}{2}$ and $F_2(x) = \dfrac{\ln(1+x^4)}{4}$ and $F_3(x) = \dfrac{\ln(1+x^6)}{6}$. Find all of the intersections of the curves you draw.

6. Graph $G_1(x) = x^{.1}$ and $G_2(x) = \ln x$ and $G_3(x) = e^{.001x}$. I *think* there are intervals where each one of these functions is bigger than the other two. Find examples of such intervals.

7. Here are some approximate values:

$$\ln 2 \approx .7 \text{ and } \ln 10 \approx 2.3$$

Use these approximations *without writing anything else* (!!!) to explain how many decimal digits to expect in the number 2^{40}. Are you correct?

8. The following statements are true facts:

$$(10,000,000,000)^{\left(\frac{1}{10,000,000,000}\right)} \approx 1.\ 00000\ 00\underline{023}\ \underline{02585}\ \underline{09}564$$

and

$$\ln 10 \approx \underline{2.30258}\ \underline{509}29$$

Explain the amazing coincidence of the digits.

Hint Approximate e^x when x is small.