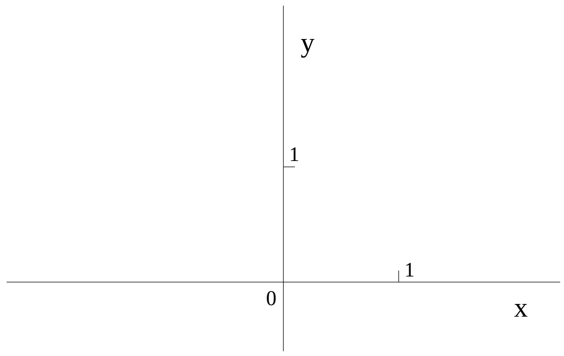
1 A

(15) 1. a) Write the definition of the derivative of a function $f(x)$ as a limit and *use this definition* to find the derivative of $f(x) = \dfrac{1}{x+2}$.

b) Find an equation for the line tangent to the curve $y = \dfrac{1}{x+2}$ at the point on the curve where $x = 1$.

c) Sketch the graph of $y = \dfrac{1}{x+2}$ *and* the tangent line requested in b) on the coordinate axes below.



(15) 2. A farmer with 750 ft of fencing wants to enclose the four sides of a rectangular region and then divide the region into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

(15) 3. Particle A moves along the positive x–axis and particle B moves along the positive y–axis.

a) If A is at the point $(a, 0)$ and B is at the point $(0, b)$, write an expression for the distance from A to B.

b) At a certain time, A is at the point $(7, 0)$ and moving *away from* the origin with a speed of 4 units/sec. At the same time B is at the point $(0, 5)$ and moving *towards* the origin at 2 units/sec. At what rate is the distance from A to B changing at that time? Are the particles moving towards each other or are they moving away from each other at that time?

(15) 4. A particle is moving with the given data. Find a formula for the position, $s(t)$, of the particle at time t if

$$a(t) = \cos t + \sin t, \;\; s(0) = 0, \;\; \text{and } v(0) = 5.$$

Here $v(t)$ is the particle's velocity at time t and $a(t)$ is its acceleration at time t.

(15) 5. Suppose that Q is the function

$$Q(x) = 5 \arcsin\left(\ln x\right).$$

a) What are the domain and range of Q? Answers should *not* be numerical approximations, but should be written if needed in terms of constants studied in calculus such as π and e.

b) Suppose $y = Q(x)$. Write a formula for x in terms of y.

(20) 6. Suppose the following is known about a twice differentiable function, H:

x	$H(x)$	$H'(x)$	$H''(x)$
1	2	0	2
2	3	6	5
3	7	3	-4
4	2	5	7

a) If w is a small number, write an approximation (the linearization of H at 2) for $H(2+w)$.

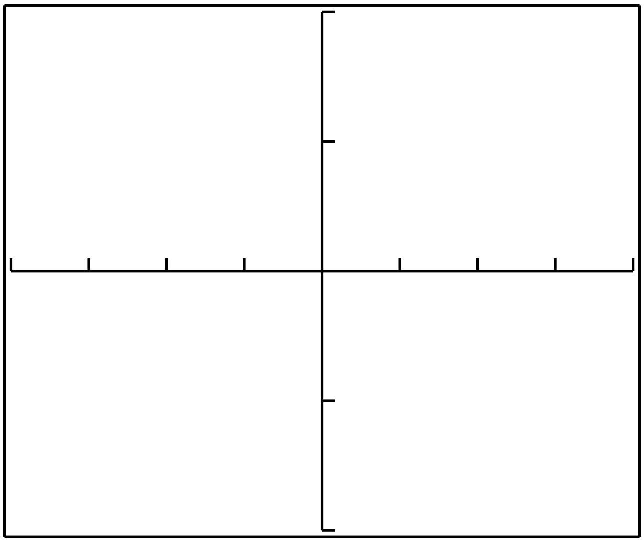
b) Is your answer in a) likely to be an underestimate of the true value of $H(2+w)$ when w is small, or an overestimate? Give a reason for your answer.

c) If $J(x) = \big(H(x)\big)^2$, compute $J(3)$, $J'(3)$, and $J''(3)$.

(20) 7. Suppose

$$f(x) = \arctan(x^2).$$

a) Sketch a graph of $y = f(x)$ in the window $-4 \le x \le 4$ and $-2 \le y \le 2$:



Now verify the geometric properties observed in the graph using calculus and algebra:

b) What are $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ and why? Explain how your results are illustrated in the graph sketched. Give the equations of any horizontal asymptotes.

c) Does the graph of $y = f(x)$ have any symmetries? Explain your answer algebraically.

This question continues on the next page.

d) Compute $f'(x)$. Find the roots of $f'(x) = 0$ algebraically. Where is $f'(x)$ positive and where is it negative? Explain what your results for f' (positive, negative, and zero) say about f itself, and how they apply to the graph sketched in a).

e) Compute $f''(x)$. Find the roots of $f''(x) = 0$ algebraically. Where is $f''(x)$ positive and where is it negative? Explain what your results for f'' (positive, negative, and zero) say about f itself, and how they apply to the graph sketched in a).

f) Does $f(x)$ have an absolute minimum on the interval $-\infty < x < +\infty$? Explain.
Does $f(x)$ have an absolute maximum on the interval $-\infty < x < +\infty$? Explain.

(15) 8. A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria.

a) Find a formula for the number $B(t)$ of bacteria after t hours if the population grows at a rate proportional to its size. Determine explicit numerical values for all the constants in your formula.

b) Find the number of bacteria after 4 hours.

c) When will the population of bacteria reach 30,000?

(18) 9. Suppose C is a constant, and that the function V is defined by the rule

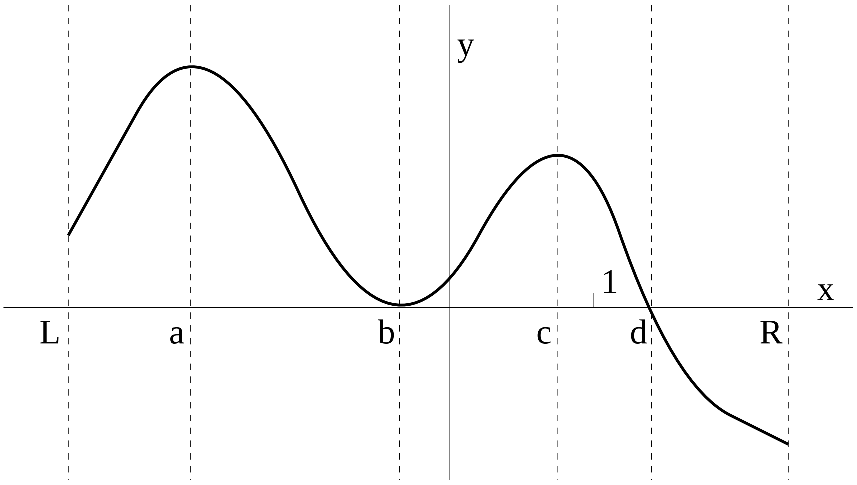
$$V(x) = \begin{cases} Cx^2 + (C^2 - C), & \text{if } x \geq 1 \\ 2x - C^2, & \text{if } x < 1 \end{cases}.$$

a) Find $\lim_{x \to 1^-} V(x)$ and $\lim_{x \to 1^+} V(x)$.

b) Which value or values of C make the function V continuous at $x = 1$? Explain in one sentence how the definition of continuity is used to determine your answers. Make a rough sketch of the curve $y = V(x)$ for *each* value of C which results in a continuous function.

c) Which value or values of C make the function V differentiable at $x = 1$? Explain in one sentence how the definition of differentiability is used to determine your answers.

(20) 10. Below is a graph of the *derivative* of the function $T(x)$:



Graph of $y = T'(x)$, the derivative of $T(x)$

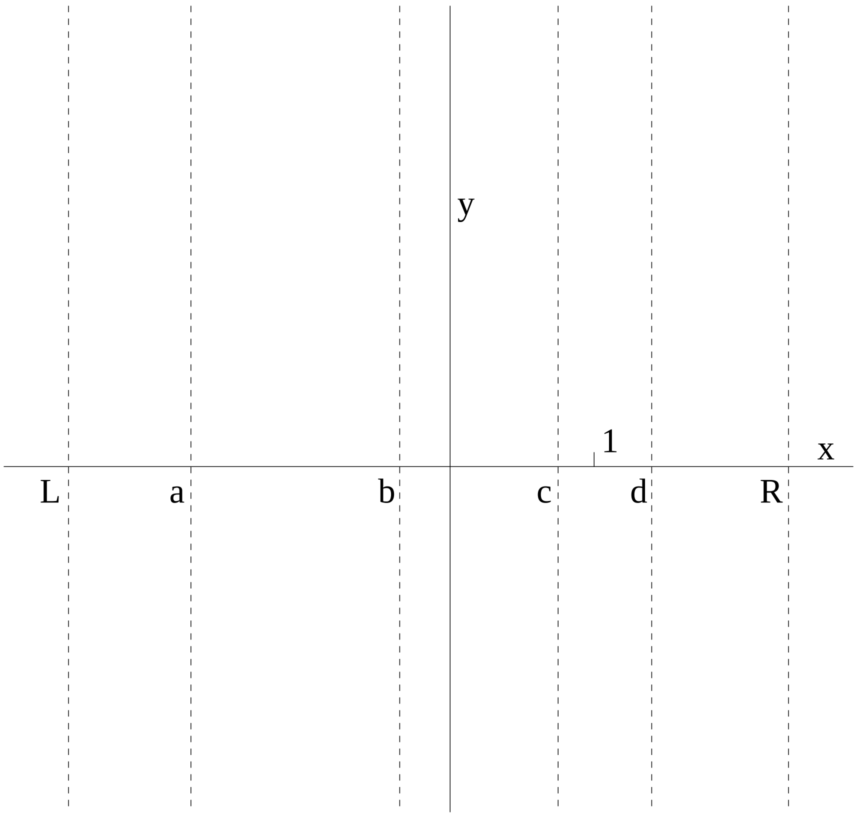
a) What are the critical numbers of T? Where does T have local extrema, and what kind of extrema are they?

b) What are the intervals in which T is increasing? What are the intervals in which T is decreasing?

This question continues on the next page.

c) Where are T's points of inflection located? What are the intervals in which T is concave up? What are the intervals in which T is concave down?

d) Suppose in addition that you know $T(0) = -1$. Use this information together with your answers to a), b) and c) to sketch a graph of $y = T(x)$ as well as you can on the coordinate axes below.

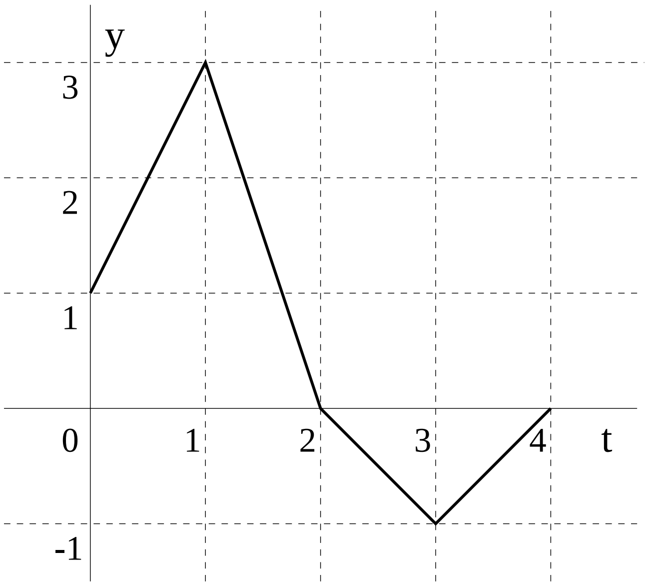


(12) 11. In each case, find the limit. Numerical answers alone are *not* sufficient. What you write should show how you get your answers. Answers should *not* be numerical approximations but should be written if needed in terms of values of functions studied in calculus.

a) $\lim\limits_{x\to+\infty}\left(\dfrac{x^3}{x^2-1}-\dfrac{x^3}{x^2+1}\right)$

b) $\lim\limits_{x\to 0}\dfrac{3^x-1}{5^x-1}$

(20) 12. Suppose the graph of f is given by:



The graph of $y = f(t)$

Let $g(x) = \int_0^x f(t)\, dt$.

a) Find $g(0)$ and $g(4)$.

b) Find $g'(1)$.

This question continues on the next page.

c) For what value of x does $g(x)$ have its maximum value?

d) Compute $\int_1^2 3x^2 - \frac{4}{x^2}\, dx$ using calculus.

A A

FINAL EXAM for MATH 151 and 153

December 18, 1995

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

Please write your instructor, course and section:

Instructor ____________ **Course** ________ **Section** ________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	15	
2	15	
3	15	
4	15	
5	15	
6	20	
7	20	
8	15	
9	18	
10	20	
11	12	
12	20	
Total Points Earned:		

A A