Calculus 151 Problems, Week 1

Please write solutions to ONE of problems (as announced in the Workshop) and hand it in at the Workshop next week. Your written solution should follow the guidelines in the sheet 'Writing up workshop problems' that you received at the workshop. You may discuss the problems with other students and with your instructors, but the written work you hand in must be your own. You should use your graphing calculator as an experimental tool whenever possible. For example, you can check particular cases of inequalities, or compare the graphs of functions this way.

DO NOT TRY TO WORK OUT THE SOLUTIONS ON THIS PIECE OF PAPER. Bring a notebook to workshop to record your calculations during the workshop. These notes can then be the starting-point (but not the finishing point) for the writeup that you hand in.

1. A rectangular box has a square bottom, no top, and a volume of 2 cubic meters.

(a) Let x be the length of a side of the base and let h be the height of the box. Make a sketch of the box, with the dimensions labeled, and find an equation relating x and h.

(b) Let y be the surface area of the box. Find a formula for y as a function of x.

(c) Use your calculator to graph the function y. What happens to the graph when x is near zero? What happens when x is large? Explain by drawing pictures of the box in these cases.

2. Suppose $f(x) = x^2 - 4$, $g(x) = 3x$, and $h(x) = \sqrt{x}$.

a) Write out formulas for the following functions (watch the punctuation):

$$y_1 = f(x)g(x)h(x), \quad y_2 = f(g(x))h(x), \quad y_3 = f(g(h(x))), \quad y_4 = h(g(f(x)))$$

Find the the domains (admissible x values) for y_1, y_2, y_3, and y_4.

b) Use your calculator to plot the graphs of the functions y_3 and y_4. Compare the domains you found in a) with the domains in the graphs that your calculator plots. Are they *exactly* the same? Are they *approximately* the same?

3. a) Use your calculator to obtain the graph of $y = \big|x - |x-3|\big|$.

b) Now obtain a "piecemeal" definition of the function whose graph you obtained with your calculator in a). *Do not* use the absolute value function. The graph may help to answer this question, but you should justify your answer algebraically with a case-by-case argument from the equation for y (of the sort "when $x \geq 3$ then y is given by the formula ...").

(OVER)

4. A common way to obtain a family of functions $f_0(x)$, $f_1(x)$, $f_2(x)$, ..., $f_n(x)$, ... is by a *recusive* formula. For example, start with $f_0(x) = 3x^2$ and define

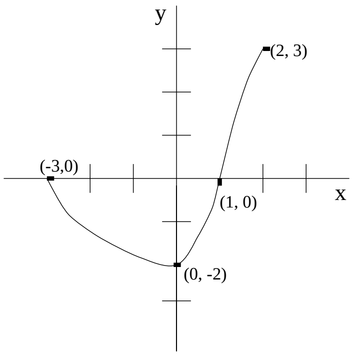
$$f_n(x) = f_0(f_{n-1}(x))$$

for $n = 1, 2, 3, \ldots$.

a) Calculate $f_1(x)$, $f_2(x)$ and $f_3(x)$.

b) Find an *explicit* formula for $f_n(x)$ in terms of n and x. First guess a formula, based on your evidence from a), and then check that your formula satisfies the recursive definition. (This is the method of 'proof by mathematical induction'.)

5. Here is a graph of $y = A(x)$:



Answer the following WITHOUT using a graphing calculator (in fact, it is IMPOSSIBLE to use your calculator since you don't have a formula for $A(x)$)
a) What are the domain and range of A?

b) Suppose B is the function defined by $B(x) = A(x) + 1$. Sketch the graph of B as well as you can. What are the domain and range of B?

c) Suppose C is the function defined by $C(x) = A(x)^2$. Sketch the graph of C as well as you can. What are the domain and range of C?

d) Suppose D is the function defined by $D(x) = \dfrac{1}{A(x)}$. Sketch the graph of D as well as you can. What are the domain and range of D?