Calculus 151 Problems, Week 2

1. A tasty bug is at point $(x, 0)$ on the x-axis and is being watched by a sparrow perched at the point $(2, 5)$ and a woodpecker who is at the point $(-1, 7)$.

a) Find a formula for the distance from the sparrow to the bug, and a formula for the distance from the woodpecker to the bug.

b) For what values of x is the bug nearer the sparrow? Give an algebraic solution and a graphical solution.

2. Suppose $f(x) = 2x^2 - x$, $g(x) = \sqrt{3x+2}$.

a) Find a formula for the function $f(g(x))$, determine its natural domain, and graph it.

b) Find a formula for the function $g(f(x))$, determine its natural domain, and graph it.

3. Let $f_n(x) = (x^n)2^{-x}$.

a) Find the graphs of $f_n(x)$ for $0 \le x \le 10$ and $n = 1, 2, 3$. You will have to adjust the viewing window to see the graph. Describe how the graphs change as n increases. What features stay the same?

b) Find the x coordinate $x_{\max}$ of the highest point of the graph for $n = 1, 2, 3$. Plot $x_{\max}$ as a function of n.

c) Without using your calculator, make a rough sketch of what you think graph of $f_5(x)$ looks like, and what the x coordinate of the highest point is, based on the evidence from (a) and (b). Then test your guess by actually generating the graph with your calculator.

4. (a) Graph the function $y = 2^x$ in the window $-1 \le x \le 1$. On the same axes draw the secant lines through the points $(x, 2^x)$ and $(0, 1)$ for $x = 0.5$ and $x = 0.1$.

(b) Let $m(x) = (2^x - 1)/x$ be the slope of the secant line to the graph in (a). Graph $m(x)$ for $-1 \le x < 0$ and $0 < x \le 1$. Give a table of values of $m(x)$ for $x = -.02, -.01, .01, .02$. (Use the TABLE key for the TI-82 or the EVAL key for the TI-85.)

(c) Determine $\lim\limits_{x \to 0} m(x)$ correct to five decimal places by using a smaller ΔTbl in the table setup. Later in the term we will show that the limit is $\ln 2$. Check this numerically with your calculator.

5. Let $f(x) = (\tan x - x)/x^3$ for $x \ne 0$.

(a) Graph $y = f(x)$ in the window $-0.5 \le x \le 0.5$. Does your graph suggest that $\lim_{x\to 0} f(x)$ exists? What seems to be the numerical value of the limit?

(b) Graph $y = f(x)$ in the window $-10^{-5} \le x \le 10^5$. Do you still think that $\lim_{x\to 0} f(x)$ exists?

(In fact the limit does exist and the value of the limit could be correctly predicted in (a), but you can't *prove* that the limit exists just by ZOOMing in on the graph. We will find this limit later using l'Hospital's rule.)