Calculus 151 Problems, Week 4

1. Let $f(x) = \sqrt{x^2 + 3x + 1} - x$.

a) Graph $f(x)$ for $0 \leq x \leq 10$.

b) Use the TABLE key (or FEVAL on the TI-85) to investigate $f(x)$ for large x (set TblMin= 1000 and ΔTbl = 1000). Do you think that $\lim_{x\to+\infty} f(x)$ exists? If you think the limit exists as a finite number, what is it?

c) Use algebra and limit theorems to prove your guess from b).

2. Determine whether each of the following statements is true or false. If you claim that the statement is true, then justify your claim by an appropriate limit law. If you claim that the statement is false, then give an example.

a) If $\lim_{x\to 0} f(x) = 3$ and $\lim_{x\to 0} g(x) = 3$, then

$$\lim_{x\to 0} \frac{f(x)^2 - g(x)^2}{f(x) - g(x)} = 6.$$

b) If $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to\infty}(f(x) - g(x)) = 0$.

c) If $f(x) > 1$ for all $x \neq 0$ and $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x) > 1$.

3. A rational function is a quotient of two polynomials, such as

$$\frac{(3x+7)(6x-1)}{(x^2+6)(x-11)}.$$

Find a rational function $R(x)$ satisfying *all* of the following properties:

i) The natural domain of $R(x)$ is all real numbers *except* $x = 1$

ii) $\lim_{x\to\infty} R(x) = 5$.

iii) $R(3) = 0$.

Sketch a graph of $y = R(x)$. Is there more than one possible choice for $R(x)$?

4. a) Draw a diagram showing two perpendicular lines that intersect on the y-axis and are both tangent to the parabola $y = x^2$.

b) Find the coordinates of the two points on the parabola where the lines in a) are tangent. Find the point on the y-axis where the two lines in a) intersect.

5. Let $f(x) = x/(x^2 + 1)$.

a) Graph $y = f(x)$ in the window $-3 \leq x \leq 3$ and $-1 \leq y \leq 1$. Find the highest and lowest points on the curve.

b) Calculate $f'(x)$ and use it to find an equation for the line that is tangent to the curve $y = f(x)$ at $x = 2$. Draw the line on the graph in a) to check the result.

c) Use the graph in a) to guess the x values where $f'(x) = 0$, where $f'(x)$ is largest (this is easy) and where $f'(x)$ is smallest (this is not so easy). Then graph the equation $y = f'(x)$ in the same window as in a) to check your guesses.