# Calculus 151 Problems, Week 9

1 . Find the following derivatives

(a) $\dfrac{d}{dx}\left(\left[\ln(1+x^2)\right]\left[\ln(1-x^2)\right]\right)$ and $\dfrac{d}{dx}\left(\ln\left[(1+x^2)(1-x^2)\right]\right)$

(b) $\dfrac{d}{dx}\left(x^{\cos x}\right)$ and $\dfrac{d}{dx}\left((\cos x)^x\right)$

(*Hint:* For (b), use the logarithmic differentiation trick: $y' = y(\ln y)'$)

2. The following information is a section of an *exceptionally* sloppy laboratory scientist's notebook.

| Time | # of bacteria |
|---|---|
| 9:30 AM | 12,000 |
| BLOB (Chicken soup) | 14,500 |
| 11:15 AM | 15,700 |
| 1:30 PM | BLOB (Chocolate pudding) |

Assume that the rate of growth of the bacteria is directly proportional to the number of bacteria present. Use this information to reconstruct the data as well as you can. Please use a calculator or a table and get numbers!

3. A small object of unknown temperature was placed in a large room that had the fixed temperature 30° C. After 10 minutes, the object's temperature is $-10°$ C, and after an additional 10 minutes, the object's temperature was $-5°$ C. What was the initial temperature of the object? (Assume that the temperature obeys Newton's law of cooling: the rate of change of the temperature of the object is proportional to the difference in temperature between the object and the constant room temperature.)

4. The derivatives of functions involving logs and inverse trig functions are usually simpler than the original functions. (Think of at least three examples!) For example, Let

$$f(x) = \tan^{-1} x + \ln\sqrt{\frac{1-x}{1+x}}, \qquad g(x) = \frac{-2x^2}{1-x^4}$$

.

(a) Graph $f(x)$ and $g(x)$ for $-1 < x < 1$.

(b) Show that $f'(x) = g(x)$ (*Hint:* Before differentiating, simplify the expression involving $\ln\sqrt{\cdots}$).

(c) Show that the result in (b) is consistent with the graphs from (a).

(d) What happens to the graphs of $f(x)$ and $g(x)$ if you change the graphing window to $-2 < x < 2$? Can you think of another pair of functions $f(x)$ and $g(x) = f'(x)$ for which the natural domain of $g(x)$ is bigger than that of $f(x)$?

5. Graph the functions $F(x) = \dfrac{4x}{x^2+1}$ and $G(x) = 2\sin(2\arctan x)$ in the same viewing window over the interval $[-5, 5]$. Give a *mathematical* explanation of what you see.