Calculus 151 Problems, Week 10

1. Find the following limits:

a) $\lim_{x \to 0} \dfrac{\sin^{-1} x - x}{x^3}$

b) $\lim_{x \to 0} \dfrac{\tan^{-1} x - x}{x^3}$

In each case, first give a table of values of the function for $x = -.1, -.01, -.001, .001, .01,$ and .1 to get an idea of the approximate value of the limit. Then apply l'Hospital's rule to find the exact value of the limit.

2. Let $f(x) = (\tan(\pi x/4))^{\tan(\pi x/2)}$.

a) Graph $y = f(x)$ in the window $0 < x < 2$ and $0 \leq y \leq 1$. Be careful to use enough parentheses when you key in the function. Why is $f(x)$ not defined at $x = 1$?

b) Use your calculator to estimate $\lim_{x \to 1} f(x)$.

c) Use l'Hospital's rule to find $\lim_{x \to 1} f(x)$.

3. Find the limits for the following indeterminate forms of the type "$\infty - \infty$".

a) $\lim_{x \to 0} \left\{ \dfrac{1}{\sin x} - \dfrac{1}{x} \right\}$

b) $\lim_{x \to 0} \left\{ \dfrac{1}{x^2} - \dfrac{1}{x} \right\}$

c) $\lim_{x \to 0} \left\{ \dfrac{1+x}{x} - \dfrac{1-x}{x} \right\}$

Note that three completely different types of behavior occur.

4. Let $f(x) = x^3 + x^2 + cx + 1$, where c is a constant.

(a) Graph $f(x)$ for $c = -1$, $c = 0$, and $c = 1$ (three different graphs). What are the similarities and differences among the graphs?

(b) For what values of c will $f(x)$ have one local maximum and one local minimum?

(c) For what values of c will $f(x)$ have no local maximum or local minimum?

5. A number x_0 is called a *fixed-point* of the function $f(x)$ if $f(x_0) = x_0$.

(a) Find the fixed points of the function $f(x) = x^2$ and $g(x) = x^3$. Find the fixed points of the function $h(x) = x^n$ where n is any positive integer greater than 3.

(b) Suppose $f'(x) < 1$ for all x in some interval $a \leq x \leq b$. Prove that $f(x)$ cannot have two fixed points in this interval. (*Hint:* Apply Rolle's theorem to the function $g(x) = x - f(x)$. What is the value of g when x is a fixed point for f?)