Calculus 151 Problems, Week 12

1. Find the dimensions of the rectangle of largest area that can be inscribed in the triangle bounded by the x-axis, the y-axis, and the line $x/3 + y/5 = 1$.

Begin your solution by drawing a picture, labeling the dimensions of the rectangle, and finding a formula for the area as a function of one of the dimensions. Then apply calculus.

2. A piece of wire $10'$ long is to be cut into two pieces. One piece will be bent into a square and the other will be bent into a circle. Let A be the sum of the areas of the square and the circle.

(a) How should the wire be cut to maximize A?

(b) How should the wire be cut to minimize A?

3. A boat leaves a dock at 2:00 p.m. and travels due south at a steady speed of 20 mi/hr. Meanwhile, another boat has been heading due east at a steady speed of 15 mi/hr and reaches the same dock at 3:00 p.m. At what time were the boats closest together?

Begin your solution by drawing a picture, labeling the coordinates of the positions of the boats, and finding a formula for the distance between the boats. Then apply calculus.

4. A $3'$ high painting is mounted on a wall so that its lower edge is $1'$ above your eye when you stand next to the wall.

a) Suppose that you stand x feet away from the wall to look at the painting. Find a formula for your viewing angle θ between the top and bottom of the painting, as a function of x. (*Hint:* $\theta = \theta_{\text{top}} - \theta_{\text{bottom}}$, where θ_{top} is the angle between the horizon and the top of the painting, and θ_{bottom} is the angle between the horizon and the bottom of the painting.)

b) Graph θ as a function of x and estimate from the graph where it has a maximum. Now use calculus to find the value of x that maximizes θ.

5. Consider the family of curves given by the equation $f(x) = x^4 + cx^2 + x$, where c is a constant.

a) Plot the curves in the window $-2 \le x \le 2$, $-2 \le y \le 2$ for the values $c = -2$, $c = -1$ and $c = 1$. Do all three curves have the same number of inflection points? Do all three curves have the same number of horizontal tangents? You will have to ZOOM in to investigate this.

b) Use calculus to determine the inflection points of the graph of $y = f(x)$ (your answer will depend upon c). What is the value of c at which the number of inflection points changes?