Calculus 151 Problems, Week 13

1. The symbol $\int f(x)\,dx$ means a function $F(x)$ whose derivative is $f(x)$; we call $F(x)$ an *antiderivative* or an *indefinite integral* of $f(x)$.

a) Evaluate five of the following six indefinite integrals.

$$\int xe^{x^2}\,dx \qquad \int (2x^9+1)^4 x^8\,dx \qquad \int 5(x+17)^{-6}\,dx$$

$$\int x^{-8}\sqrt{2x^{-9}+1}\,dx \qquad \int x\sin(\pi x^2)\,dx \qquad \int \frac{2x}{\sqrt{1-x^4}}\,dx$$

b) Modify the integral that you omitted to change it into one that you can evaluate (there are many ways to do this–find at least two such modifications).

2. a) A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 40 ft/s^2. What is the distance covered before the car comes to a stop?

b) A car braked with a constant deceleration of 40 ft/s^2 and produced skid marks measuring 160 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

3. Let the function $f(x)$ be defined by $f(x) = x$ for $0 \le x \le 1$, $f(x) = 1$ for $1 \le x \le 2$, and $f(x) = 3 - x$ for $2 \le x \le 3$.

a) Draw the graph of $y = f(x)$ for $0 \le x \le 3$.

b) Let $0 \le b \le 3$. Define $F(b)$ to be the area under the graph of $y = f(x)$ between $x = 0$ and $x = b$. Use elementary geometry to find a formula for $F(b)$. You will have to consider the cases $0 \le b \le 1$, $1 \le b \le 2$ and $2 \le b \le 3$ separately. Draw the graph of $y = F(b)$.

c) Calculate $F'(x)$ for $0 \le x \le 3$ and draw the graph of $y = F'(x)$. Have you seen this graph recently? (This is not a coincidence; it's a consequence of the Fundamental Theorem of Calculus.)

(Over)

4. Using the calculator, graph the function $f(x) = \dfrac{1}{x^2+1}$ on the interval $0 \le x \le 2$.

a) What is the largest value of $f(x)$ on the interval $0 \le x \le 2$? The smallest value?

b) Use your answers to a) and the geometric meaning of the definite integral as an area to show that

$$0.4 \le \int_0^2 \frac{1}{x^2+1}\,dx \le 2.$$

c) By cutting the interval $0 \le x \le 2$ into two pieces and repeating a) and b) for each piece, show that

$$0.7 \le \int_0^2 \frac{1}{x^2+1}\,dx \le 1.5.$$

d) Now cut the interval $0 \le x \le 2$ into four pieces and repeat a) and b) for each piece to find closer upper and lower estimates for the integral. Show the graphical intepretation.

e) Use the `fnInt(` program in your calculator to calculate the numerical value of the integral. (This program is in the `MATH` menu; see your calculator's instruction manual for examples.)

Note: By the fundamental theorem of calculus, the definite integral has the value $\arctan(2)$.