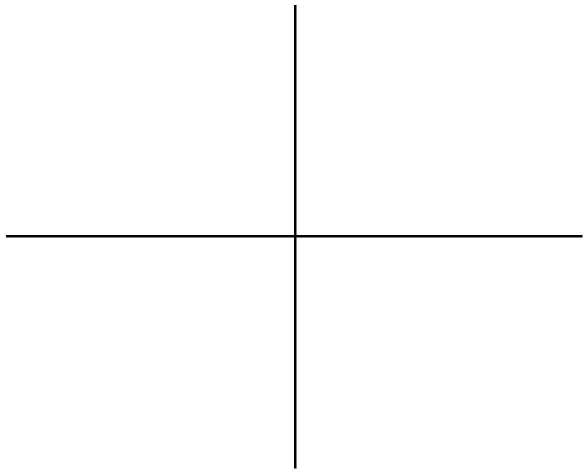
Name: ____________________

(18) 1.

(a) Sketch the region R bounded by the curves $y = x^2 + 1$ and $y = 3 - x^2$.



(b) Find the area of R. (Give exact answer, show work.)

(c) Compute the volume which results when the region R is revolved about the x-axis. (Give exact answer, show work.)

(d) Set up **but do not evaluate** an integral for the volume which results when the region R is revolved about the line $x = 2$.

 Name: ______________________

(32) 2. Integrate:

(a) $\displaystyle\int \frac{15x}{(x-4)(x^2+4)}\,dx$ (Hint: partial fractions)

(b) $\displaystyle\int \frac{7}{2x\sqrt{x^2+9}}\,dx$ (Hint: make a trigonometric substitution)

2. (continued)

(c) $\displaystyle\int_0^{\pi/4} \sin^4 x \cos^5 x \, dx$. (Hint: start by using a trigonometric identity)

Give an exact answer, which you may leave as a sum of fractions.

(d) $\displaystyle\int x^2 e^{-7x} dx$ (Hint: integration by parts)

(14) 3. (a) Solve the initial value problem

$$\frac{dy}{dx} = \frac{-3\sin 3x}{y}, \qquad y(0) = 1.$$

Express your solution y explicitly, not implicitly, as a function of x.

(b) What is the largest interval containing the point $x = 0$ on which your solution is defined?

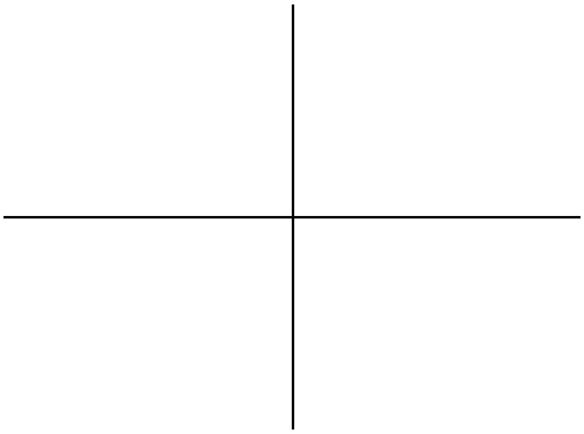
(12) 4. Find the length of the curve

$$x = e^{2t} - 2t, \quad y = 4e^{t}, \qquad 1 \leq t \leq 3.$$

Give an exact answer in terms of well-known constants such as e, π, $\sqrt{3}$, etc., **not** a decimal approximation.

(12) 5.

(a) Sketch the two curves $r = 1 + 2\sin\theta$ and $r = 4\sin\theta$, $0 \le \theta \le \pi$, labelling the x- and y-coordinates of the points of intersection.



(b) **Set up** an integral representing the area of the region outside $r = 1+2\sin\theta$ but inside $r = 4\sin\theta$.

(12) 6. The curve $x = 8 - 2t^2$, $y = \sin \pi t$, $-4 \le t \le 4$ crosses itself at the origin.

(a) What are the t-values at which it crosses the origin?

(b) Find the equations of both tangent lines at the origin.

(28) 7. For each of the following series, check the appropriate box according as the series converges absolutely, converges conditionally or diverges. In each case **state the convergence test used and show the method used.**

(a) The series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+3}$ is
- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

(b) The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^2}$ is
- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

7. (continued)

(c) The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^{1/n}}$ is
- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

(d) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1+e^n}{7^n}$ is
- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

(12) 8. How many terms of the series

$$\sum_{n=1}^{\infty} \frac{4}{n^{5/2}}$$

are required to approximate the sum of the series to within .001? Use the integral test error estimate and show your reasoning.

(6) 9. Suppose that $\{a_n\}$ is a convergent sequence and $\lim_{n\to\infty} a_n = L$. Is the sequence $\left\{ \frac{3n}{2n+5}\, a_n \right\}$ convergent, and if so, what is its limit? Show reasoning.

(12) 10. What is the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(3x)^n}{n+4}$? For which values of x does the series converge?

(6) 11. Without attempting to compute it, decide whether the improper integral

$$\int_0^{\infty} e^{-x^{12}}\, dx$$

converges or diverges. Show your reasoning.

(14) 12. (a) Use the Maclaurin series for e^x to get an infinite series for the integral $I = \int_0^1 e^{-\frac{1}{2}x^2}\,dx$. Your answer should include an expression for the n-th term of this series.

(b) Use the first three nonzero terms of your series to get an approximate value for I.

(c) Estimate the error in this approximation, showing your reasoning.

(10) 13. Approximate $\int_0^1 e^{-\frac{1}{2}x^2}\,dx$ by Simpson's Rule with $n = 4$, thus evaluating the integrand at $x = 0, .25, .5, .75$ and 1. Your answer should be an exact expression in terms of powers of e, not a decimal approximation.

(12) 14. (a) Find the second degree Taylor polynomial $T_2(x)$ for $f(x) = x^{10}$ about $a = 1$.

(b) Use this polynomial to approximate $(1.01)^{10}$.

(c) Taylor's formula tells you that the error in this approximation is between two numbers. What are they? Give exact expressions, not decimal approximations.

FINAL EXAM for MATH 152/154 May 5, 1999, 4–7 p.m.

NAME (*please print*): ____________________

SIGNATURE: ____________________

SECTION NUMBER: __________

PLEASE CIRCLE YOUR INSTRUCTOR:

Beck—Cherlin—Gundy—Han—Irvine—Levitt—Lyons—Roberts—Sahi—Speer—Weinstein

INSTRUCTIONS

Do all problems. There are 200 points and 180 minutes.

SHOW ALL YOUR WORK. Full credit may not be given for an answer alone.

You may use a TI-82, 83, 85, or 86 calculator or equivalent. You may **not** use a TI-92 or any calculator with a QWERTY keyboard. You may use **one handwritten** 8.5×11 sheet of your notes.

Some questions explicitly ask for an **exact answer** in terms of well-known constants such as e, π, $\ln 3$, $\sqrt{2}$, etc., as opposed to a **decimal approximation**. Be careful to give your answers in the appropriate form.

In your explanations, you need not re-derive facts derived in class, such as particular series expansions for familiar elementary functions, and the fact that certain particular series converge or diverge. But state these facts clearly when you use them.

Problem Number	Possible Points	Points Earned:
1	18	
2	32	
3	14	
4	12	
5	12	
6	12	
7	28	
8	12	
9	6	
10	12	
11	6	
12	14	
13	10	
14	12	
Total:	200	