Name: ____________________

Math 152, Sec. 72

First Hour Examination

Oct. 8, 1998

1. You may use one page of notes and any standard calculator without a QWERTY keypad on this examination. No other materials may be used.

2. The distribution of points for the problems is given below. In problems with several parts, unless otherwise stated, all parts count equally.

3. In all problems, show your work. Credit may not be given for an answer alone.

Problem	Points
1	8
2	8
3	16
4	8
5	8
6	24
7	12
8	8
9	8
Total	100

1. Find the (finite) area bounded by the parabola $y = 2 - x^2$ and the line $y = -x$.

2. A solid has as its base the region in the first quadrant, bounded by the x-axis, the y-axis and the parabola $y = 1 - x^2$. A cross-section of the solid perpendicular to the x-axis is a square. Find the volume of the solid.

3. A (finite) region R in the first quadrant is bounded by the curves: $y = x^{1/2}$ and $y = x^3$.

Find the volume of the solid that results when R is rotated:

(a) about the x-axis;

(b) about the y-axis.

4. Find the average value of the function $f(x) = \sin^2(x)$ on the interval $[0, \pi]$. Show how the integral is calculated.

5. Calculate the following integral giving an exact answer in terms of mathematical constants such as π and e, not a numerical approximation:

$$\int_1^e (\ln x)^2 \, dx$$

6. Calculate the following integrals, showing your work:

(a) $\displaystyle\int \frac{3x^2 + x}{(x-1)(x^2+1)}\,dx$

(b) $\displaystyle\int \frac{x^3\,dx}{\sqrt{1-x^2}}$

(c) $\displaystyle\int \frac{dx}{e^x + 1}$

7. Determine which of the following integrals converge and which diverge, giving your reasons and the value of those that are convergent.

(a) $\displaystyle\int_0^\infty xe^{-2x}\,dx$

(b) $\displaystyle\int_0^{\infty} \frac{x^3\,dx}{x^4+1}$

8. The integral $\displaystyle\int_0^1 \sin(x^3)\,dx$ is approximated, using the Midpoint Rule, by dividing $[0,1]$ into n segments of equal length. How large should n be in order to guarantee that the error is at most 10^{-6}?

9. A region R in the first quadrant is bounded by the parabolas $y = x^2$ and $x = y^2$. Find to four place accuracy a number c so that the vertical line $x = c$ divides the region R into two subregions of equal area.

For 3 points extra credit, find c exactly.