Name: ____________________

Math 152, Sec. 72

Second Hour Examination

Nov. 19, 1998

1. You may use one page of notes and any standard calculator without a QWERTY keypad on this examination. No other materials may be used.

2. The distribution of points for the problems is given below. In problems with several parts, unless otherwise stated, all parts count equally.

3. In all problems, show your work. Credit may not be given for an answer alone.

Problem	Points
1	7
2	7
3	8
4	5
5	15
6	14
7	16
8(a)	4
8(b)	8
9	8
10	8
Total	100

1. Find a solution to the differential equation $\frac{dy}{dx} = \frac{2xy}{x^2+1}$ that satisfies the initial condition $y(1) = 4$. In the answer express y explicitly as a function of x.

2. Calculate the arc length of the curve $y = 2x^{3/2}$ from $x = 0$ to $x = 1$. Give an exact answer such as e or $\sqrt{2}$, not a numerical approximation.

3. Evaluate the following sequential limits. Give exact answers, not decimal approximations.

(a) $\lim_{n\to\infty} \frac{\sqrt{2n^2+3n+1}}{n}$

(b) $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^n$

4. Determine the sum of the following infinite series. Express the answer as a rational number, i.e., as the quotient of two integers.

$$\sum_{n=0}^{\infty} \frac{2^n+(-1)^n}{5^n}$$

5. Test each of the following series for convergence or divergence. State which test has been used and explain why it applies.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^n}{n\, e^n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

6. Each of the following series converges. How many terms must be included to calculate the sum of each series with error at most $1/10^8$? Explain your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

7. Determine the radius and interval of convergence of each of the following power series. In addition, determine whether each series is absolutely or conditionally convergent at the boundary points of the interval of convergence.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{n^2 4^n}$

8. Suppose that n is a positive integer.

(a) Find the n-th Taylor polynomial, $T_n(x)$, of $f(x) = e^x$ centered at $a = 0$.

(b) Suppose that $T_n(x)$ is used to approximate e^x at all points of the interval $[0, 2]$. Find n so that the error in using the approximation is at most $1/10^3$ at all points of the interval $[0, 2]$. Explain your answer.

9. Calculate the fourth Taylor polynomial of $f(x) = \ln(1+x)$ centered at $a = 0$. Show your work.

10. Determine the Taylor expansion at $a = 0$ of the function:

$$F(x) = \int_0^x \cos(t^2)\, dt$$