Math 152 Sample exam #1

1. Compute in terms of values of functions used in calculus and (where possible) named constants such as e and π:

a) $\displaystyle\int_0^a x\sqrt{x^2+a^2}\,dx$ (here a is a positive constant)

b) $\displaystyle\int_0^\pi (\sin x)^5\ dx$

c) $\displaystyle\int_1^e x^2(\ln x)^2\ dx$

d) $\displaystyle\int_0^1 \frac{x^2-x+3}{(x+1)(x^2+1)}\ dx$

2. a) Sketch the area in the first quadrant enclosed by the curves $y=x^4$ and $x=y^2$.

b) Compute this area.

c) Compute the volume which results when the area described above is revolved about the $x-$axis.

d) Compute the volume which results when the area described above is revolved about the $y-$axis.

3. The base of a solid is the ellipse $x^2+3y^2=1$. Cross-sections of the solid perpendicular to the $y-$axis are squares. What is the volume of the solid?

4. The area under the first arch of $y=\sin x$ is divided in half by the straight line $y=mx$. What is m's approximate value?

5. A bucket that weighs 4 lb and a rope of negligible weight are used to draw water from a well that is 80 ft deep. The bucket starts with 40 lb of water and is pulled up at a rate of 2 ft/sec but water leaks out of a hole in the bucket at a rate of 0.2 lb/sec. Find the work done in pulling the bucket to the top of the well.

6. Suppose $f(x)$ is a function with the following properties:

i) $f(0)=1$.

ii) The average value of the function $f(x)$ over any interval $[a,b]$ is $3\left(\dfrac{f(b)-f(a)}{b-a}\right)$.

Find a formula for $f(x)$.

7. Find the average value of the function $f(x)=(\sin x)^2$ on the interval $0\le x\le\pi$.

8. a) Compute $\displaystyle\int\frac{\sqrt{x^2-2}}{x}\ dx$.

b) Use your result from a) to compute $\displaystyle\int_3^4 \frac{\sqrt{x^2-2}}{x}\,dx$ in terms of values of familiar functions. Use your calculator to evaluate these functions and report an approximate value for the definite integral.

c) Use one of the numerical methods studied in this course to approximately evaluate $\displaystyle\int_3^4 \frac{\sqrt{x^2-2}}{x}\,dx$. Be sure to specify which method and how you use the method as precisely as possible.

9. How large should n be to approximate $\displaystyle\int_0^1 \cos\left(x^2\right)\,dx$ using the Trapezoidal Rule with an error of at most 10^{-5}?

10. a) Find a substitution u that shows $\displaystyle\int 2\,(\sec x)^2 \tan x\,dx = (\tan x)^2 + C$.

b) Find a different substitution u that shows $\displaystyle\int 2\,(\sec x)^2 \tan x\,dx = (\sec x)^2 + C$.

c) Can you conclude from a) and b) that the functions $(\sec x)^2$ and $(\tan x)^2$ are equal? If you cannot, describe and explain what you *can* conclude.

11. Evaluate $\displaystyle\int_0^b \sin(5x)\sin(8x)\,dx$ in terms of $\sin(5b)$, $\cos(5b)$, $\sin(8b)$, and $\cos(8b)$.

12. a) Find the length of the curve given by $f(x) = \frac{4}{3}x^{3/2}$ from $x = 0$ to $x = 2$.

b) Find a value of x_0 between 0 and 2 so that the length of this curve $f(x) = \frac{4}{3}x^{3/2}$ from $x = 0$ to $x = x_0$ will equal the length from $x = x_0$ to $x = 2$.

13. Determine whether each of the following is convergent or divergent, and evaluate those that are convergent. Be sure to show your work and explain your reasoning.

a) $\displaystyle\int_1^e \frac{1}{x\,(\ln x)^2}\,dx$

b) $\displaystyle\int_e^\infty \frac{1}{x\,(\ln x)^2}\,dx$

c) $\displaystyle\int_2^4 \frac{1}{\sqrt{x-2}}\,dx$

d) $\displaystyle\int_0^{\pi/4} \frac{1}{(x^2+1)\arctan x}\,dx$

14. a) Verify that any function of the form $y = a + b\ln x$ (with a and b arbitrary constants) satisfies the differential equation

$$xy'' + y' = 0 \ .$$

b) Find the particular solution of the differential equation above which satisfies the initial conditions $y(2) = 0$ and $y'(2) = \frac{1}{2}$.

c) Find the particular solution of the differential equation above which satisfies the initial conditions $y(2) = 0$ and $y'(2) = 1$.

d) Show the plots of the two particular solutions that you obtained in b) and c) above, on the window $[1, 3]$ (both plots on the same graph), and interpret in this graph the meaning of the initial conditions.

15. Consider the region R bounded by the $x-$axis, the $y-$axis, and the curve $y = e^{-x}$.

a) Suppose R is rotated about the $x-$axis. Find the volume of the solid which is generated.

b) Is the surface area of the solid described in a) finite or infinite?

Hint: remember, to show that an improper integral converges or diverges, you don't necessarily have to compute the integral.