Math 152 Sample exam #2

1. Determine whether any of the following sequences converge, and find the limits when they exist. Explain your answers.

a) $\left\{\dfrac{\ln(2+e^n)}{3n}\right\}$ b) $\left\{\dfrac{\ln\left(\ln\left(n^4\right)\right)}{(\ln\, n)^3}\right\}$ c) $\left\{\sqrt[n]{6n^8}\right\}$

2. Determine whether any of the following series converge, and find their sums when they exist. Explain your answers.

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n+4^n}{5^n}$ c) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{(-1)^n}{n^2}\right)$

3. Do the following series converge? Do they converge absolutely? Do they converge conditionally? In each case that the series converges, find a partial sum which is within .001 of the sum of the series. Explain your answers.

a) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$ b) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^2\ln n}$ c) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n(1+n^2)}{n\ln n}$

4. For which x's do the following series converge? Be as precise as you can, and explain your answers. Do the series involved converge absolutely or conditionally?

a) $\displaystyle\sum_{n=1}^{\infty}\frac{n^3x^n}{3^n}$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^{1/4}}$

5. Write the value of

$$\int_0^{.8}\sin(x^4)\,dx$$

as a series. Find a partial sum of the series which is within .0001 of the value of the definite integral.

6. Find

$$\lim_{x\to 0}\frac{\left(e^{3x}-1\right)^4}{\cos(x^2)-1}$$

and explain the process used.

7. Find the Taylor series centered at 0 (the Maclaurin series) for the following functions:

a) $\dfrac{1}{1+x^3}$ b) $x\sin(2x)$ c) $\dfrac{\ln(1+x^2)}{x}$

8. Find the terms up to and including the x^4 term of the Taylor series centered at 0 (the Maclaurin series) for the following functions:

a) $\dfrac{\sin x}{1+x}$ b) $\dfrac{e^x-e^{-x}}{2}$ c) $\left(1+x^{10}\right)^{1/7}$ d) $e^{(x^2)}$

9. Find a polynomial $P(x)$ so that the difference

$$|P(x) - \cos x|$$

will be *less than* 10^{-3} in the interval $[-2, 2]$.

10. Describe how to approximate $e^{-1/2}$ with an error of size at most .0001 using a Taylor polynomial. You must be sure to give an explicit error estimate, and to write an explicit partial sum which correctly approximates $e^{-1/2}$. (Do the arithmetic needed for the error estimate; do not do the arithmetic involved in the partial sum!)

11. Suppose that f is a function whose fourth derivative satisfies the inequality

$$\left| f^{(4)}(x) \right| \leq \frac{1}{3 + 5x^2}$$

for all x and that $T_3(x)$ is the third degree Taylor polynomial for f centered at 0. Find an estimate for the error obtained when using $T_3(2)$ in place of $f(2)$.

12. What is the second degree Taylor polynomial centered at 0, $T_2(x)$, for the function $f(x) = (1+x)^{1/4}$? Use Taylor's Theorem to estimate the difference between $T_2(.2)$ and $f(.2)$.